Cellular Automata: Examples of Left and Right Kan Extensions

<u>Luidnel MAIGNAN</u>
Antoine SPICHER
Alexandre FERNANDEZ
LACL
Univ Paris-Est Créteil

Kan Extensions

$$\begin{tikzcd} & B \arrow[dr, bend left, "H"] & \\ A \arrow[ur, "F"] \arrow[rr, "G"'] & \arrow[u, phantom, "\theta\Uparrow"] & C \end{tikzcd}$$

$\theta : G \Rightarrow H \circ F$

universal

$\langle H, \theta \rangle = \mathrm{Lan}_F G$

left

Kan Extensions

$$
\begin{tikzcd}
B \arrow[dr, "H"] \arrow[dr, bend left=80, "H'"] \arrow[dr, phantom, bend left=40, "\Uparrow"] & \\
A \arrow[u, "F"] \arrow[r, "G"'] \arrow[ur, phantom, "\theta \Uparrow \quad \theta'"] & C
\end{tikzcd}
$$

universal

$\langle H, \theta \rangle = \mathrm{Lan}_F G$

left

$\theta : G \Rightarrow H \circ F$

Kan Extensions

$$\begin{tikzcd}[row sep=large, column sep=large]
B \arrow[dr, "H"] \arrow[dr, bend left=70, "H'"] \arrow[dr, phantom, bend left=35, "\Uparrow\varphi"] \\
A \arrow[u, "F"] \arrow[r, "G"'] \arrow[ur, phantom, "\Uparrow\theta\ \Uparrow\theta'"] & C
\end{tikzcd}$$

$\theta : G \Rightarrow H \circ F$

universal

$\langle H, \theta \rangle = \mathrm{Lan}_F G$

left

$$\boxed{\begin{array}{c}\text{unique } \varphi \text{ s.t.}\\ \theta' = (\varphi \circ F) \bullet \theta\end{array}}$$

Kan Extensions

$$\begin{tikzcd} B \arrow[dr, "H"] & \\ A \arrow[u, "F"] \arrow[r, "G"'] \arrow[u, phantom, shift left=4ex, "\theta\Uparrow"] & C \end{tikzcd}$$

$$\theta : G \Rightarrow H \circ F$$

universal

$$\langle H, \theta \rangle = \mathrm{Lan}_F G$$

left

Kan Extensions

$$\begin{tikzcd} B \arrow[dr, bend left, "H"] & \\ A \arrow[u, "F"] \arrow[r, "G"'] \arrow[ur, phantom, "\theta \Downarrow"] & C \end{tikzcd}$$

$\theta : H \circ F \Rightarrow G$

universal

$\langle H, \theta \rangle = \mathrm{Ran}_F G$

right

Kan Extensions

$$
\begin{tikzcd}
B \arrow[dr, "H"] \arrow[dr, bend left=70, "H'"] \arrow[dr, phantom, bend left=35, "\overset{\varphi}{\Downarrow}"] & \\
A \arrow[u, "F"] \arrow[r, "G"'] \arrow[ur, phantom, "\theta \Downarrow \ \Downarrow \theta'"] & C
\end{tikzcd}
$$

universal

$$\langle H, \theta \rangle = \mathrm{Ran}_F G$$

right

$$\theta : H \circ F \Rightarrow G$$

$$\boxed{\begin{array}{c}\text{unique } \varphi \text{ s.t.} \\ \theta' = \theta \bullet (\varphi \circ F)\end{array}}$$

Kan Extensions

2-arrows: left – right

$$
\begin{tikzcd}
B \arrow[dr, "H"] \arrow[dr, bend left=70, "H'"] \arrow[dr, phantom, bend left=35, "\overset{\varphi}{\Downarrow}"] & \\
A \arrow[u, "F"] \arrow[r, "G"'] \arrow[ur, phantom, "\theta \Downarrow \ \Downarrow \theta'"] & C
\end{tikzcd}
$$

universal

$$\langle H, \theta \rangle = \mathrm{Ran}_F G$$

right

$$\theta : H \circ F \Rightarrow G$$

$$\boxed{\begin{array}{c}\text{unique } \varphi \text{ s.t.} \\ \theta' = \theta \bullet (\varphi \circ F)\end{array}}$$

Kan Extensions

2-arrows: left – right
1-arrows: extension – lifting

$$\begin{tikzcd}
B \arrow[dr, "H"] \arrow[dr, bend left=60, "H'"] \arrow[dr, phantom, bend left=30, "\varphi\Downarrow"] \\
A \arrow[u, "F"] \arrow[r, "G"'] \arrow[ur, phantom, "\theta\Downarrow\ \Downarrow\theta'"] & C
\end{tikzcd}$$

$\theta : H \circ F \Rightarrow G$

universal

$\langle H, \theta \rangle = \mathrm{Ran}_F G$

right

$$\boxed{\text{unique } \varphi \text{ s.t. } \theta' = \theta \bullet (\varphi \circ F)}$$

Kan Extensions

$$
\begin{tikzcd}
B \\
A \arrow[u, "F"]
\end{tikzcd}
$$

Kan Extensions

$$\begin{tikzcd} & B \\ C \arrow[ur, -, squiggly] \arrow[dr, -, squiggly] & \\ & A \arrow[uu, "F"] \end{tikzcd}$$

Kan Extensions

$$
\begin{tikzcd}
B \\
A \arrow[u, "F"] \arrow[r, "G"'] & C
\end{tikzcd}
$$

Kan Extensions

$$\begin{tikzcd} B \arrow[dr, "H"] & \\ A \arrow[u, "F"] \arrow[r, "G"'] \arrow[ur, phantom, "="] & C \end{tikzcd}$$

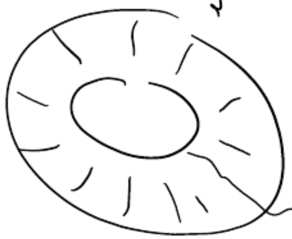
$$H \circ F = G$$

~~An~~ Extensions

$$
\begin{tikzcd}
B \arrow[dr, "H"] & \\
A \arrow[u, "F"] \arrow[r, "G"'] \arrow[ur, phantom, "="] & C
\end{tikzcd}
$$

$$H \circ F = G$$

~~An~~ Extensions

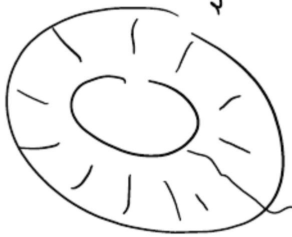


$B \backslash A$

$$\begin{tikzcd} & B \arrow[dr, "H"] & \\ & A \arrow[u, "F"] \arrow[r, "G"'] \arrow[ur, phantom, "="] & C \end{tikzcd}$$

$$H \circ F = G$$

~~Kan~~ Extensions

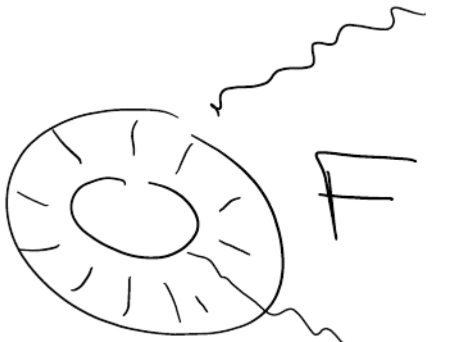


$B \setminus A$

$$\begin{tikzcd} & B \arrow[dr, "H"] \arrow[dr, bend left=60, "H'"] & \\ A \arrow[ur, "F"] \arrow[rr, "G"'] \arrow[rr, phantom, shift left=4ex, "="] & & C \end{tikzcd}$$

$$H \circ F = G$$

! Kan Extensions



B\A

$$\begin{tikzcd}
B \arrow[dr, "H"] \arrow[dr, bend left=60, "H'"] & \\
A \arrow[u, "F"] \arrow[r, "G"'] \arrow[ur, phantom, "="] & C
\end{tikzcd}$$

$$H \circ F = G$$

Kan Extensions

$B \setminus A$

$$\begin{tikzcd}
B \arrow[dr, "H"] \arrow[dr, bend left=60, "H'"] \arrow[dr, phantom, bend left=30, "\Uparrow \varphi"] \\
A \arrow[u, "F"] \arrow[r, "G"'] \arrow[ur, phantom, "="] & C
\end{tikzcd}$$

$$H \circ F = G$$

Kan Extensions

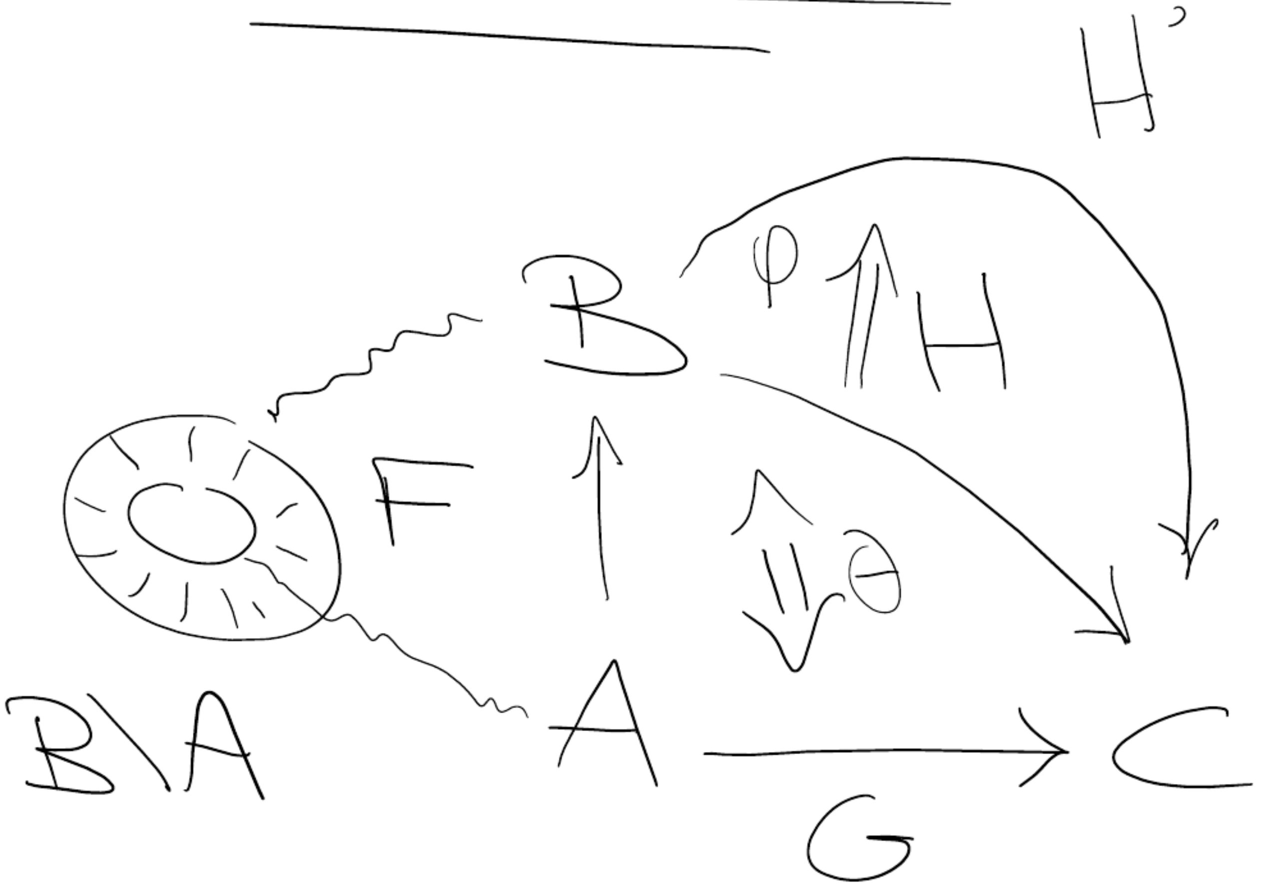
optimal H!

→ minimal

→ maximal

$H \circ F = G$

Kan Extensions



optimal H!

→ minimal

→ maximal

$H \circ F \Leftrightarrow G$

Kan Extensions

optimal H!

→ minimal

→ maximal

$H \circ F \Leftrightarrow G$

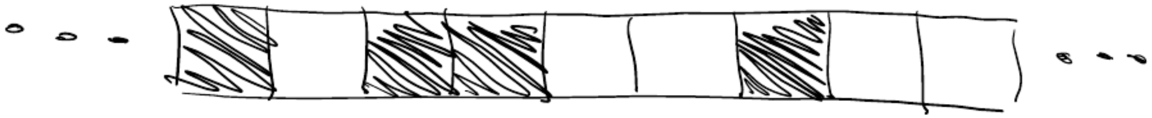
So, of course "everything is a Kan extension"

A first Cellular Automaton

"traffic rule"

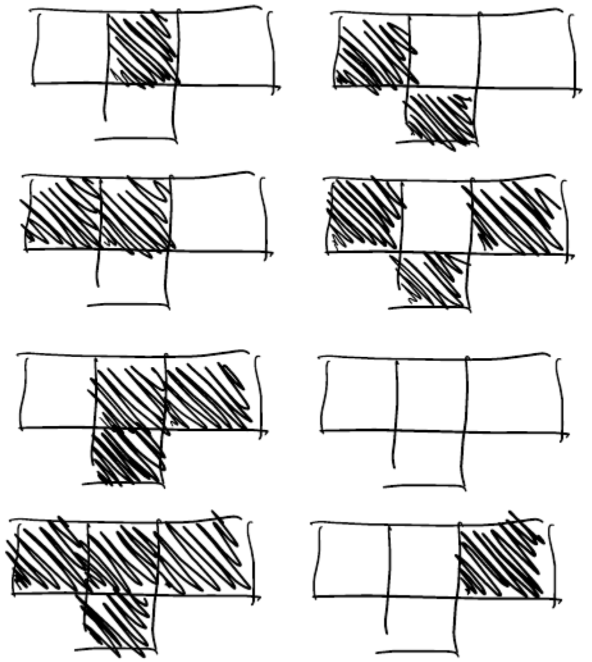
A first Cellular Automaton

"traffic rule"



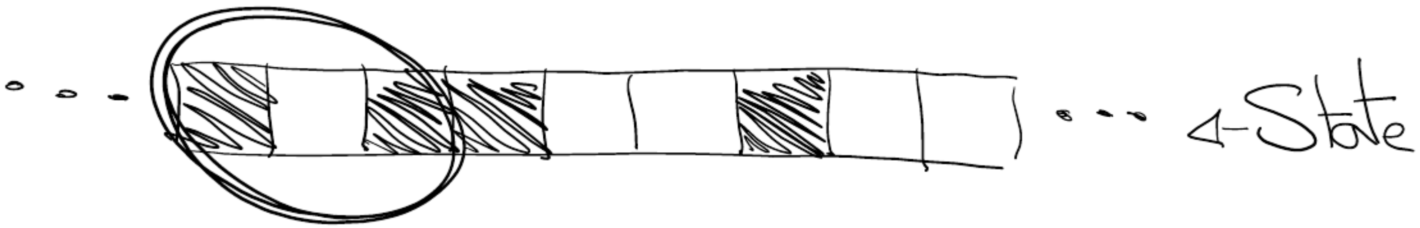
4-State

Evolution Rule

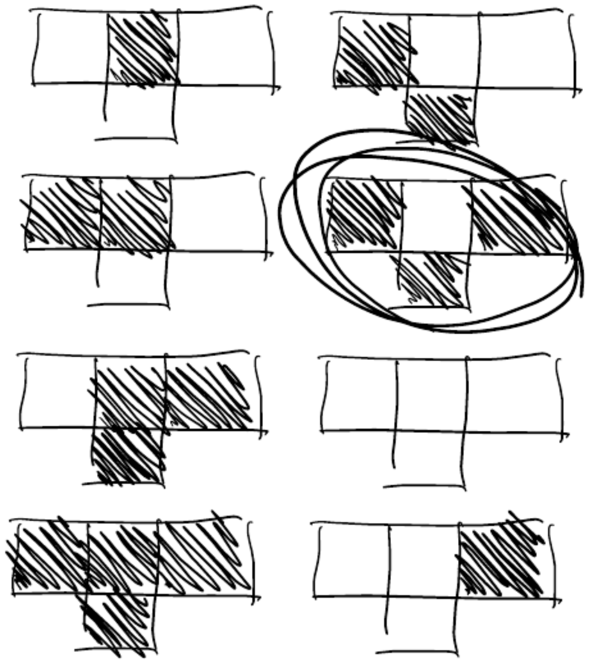


A first Cellular Automaton

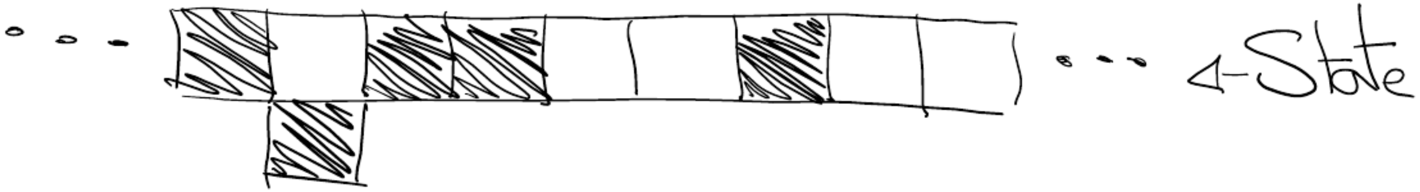
"traffic rule"



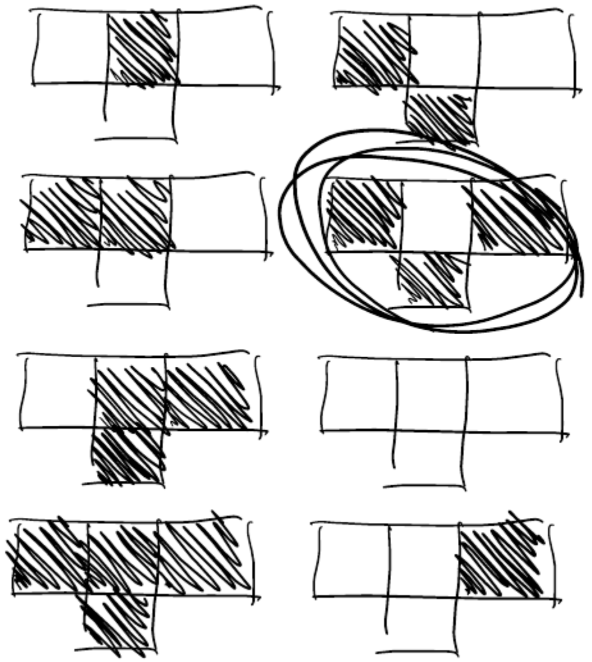
Evolution Rule



A first Cellular Automaton
"traffic rule"

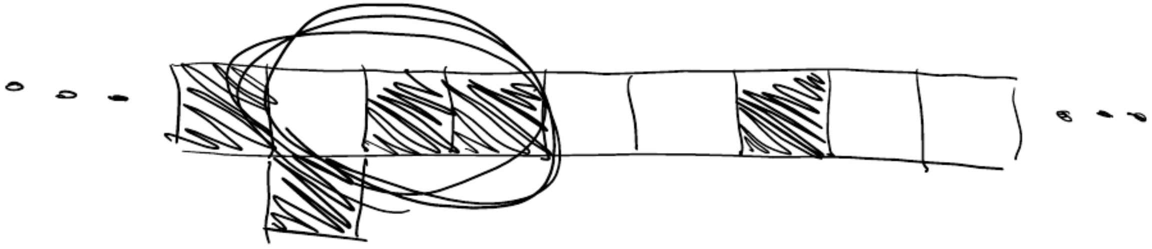


Evolution Rule



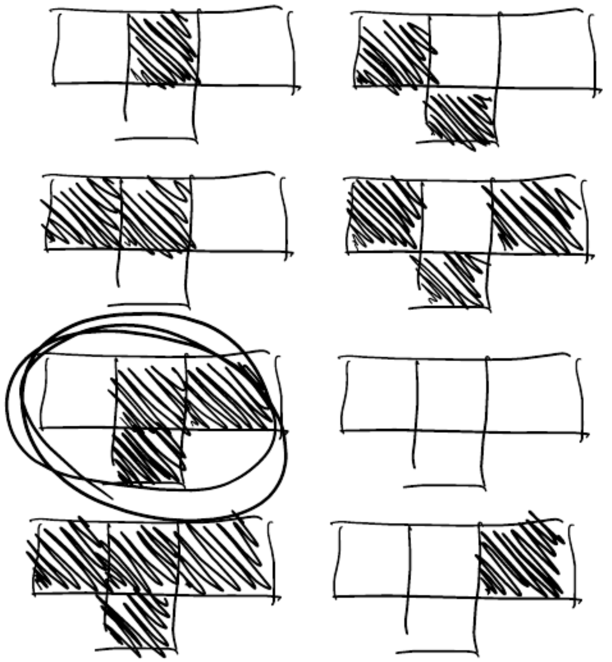
A first Cellular Automaton

"traffic rule"



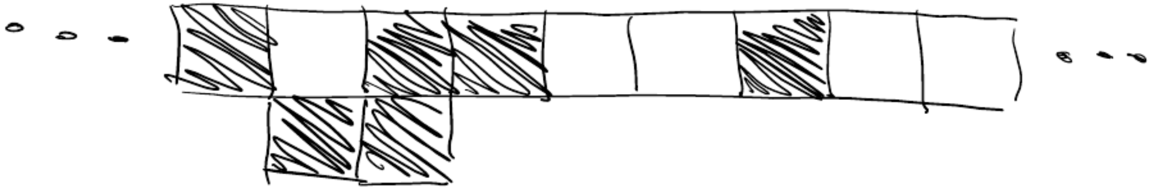
... 4-State

Evolution Rule



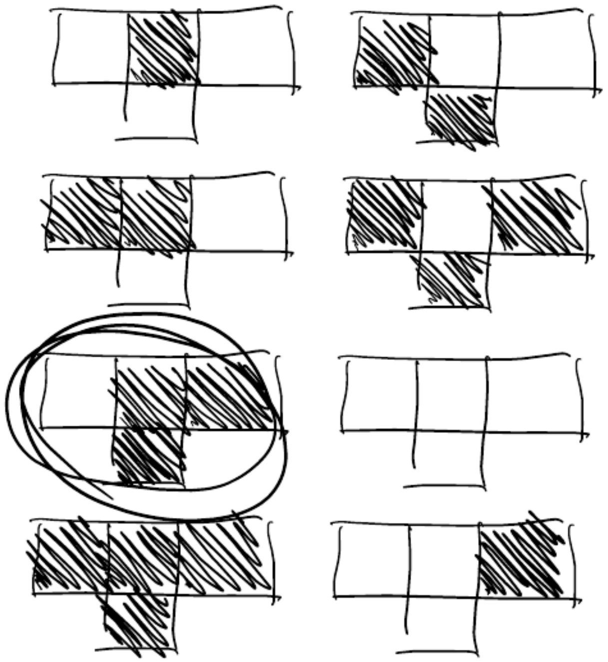
A first Cellular Automaton

"traffic rule"



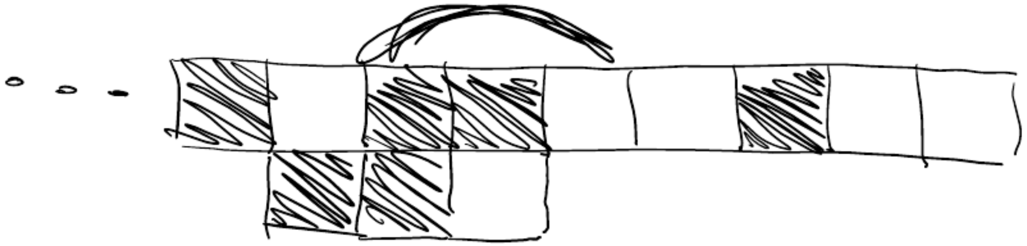
1-State

Evolution Rule



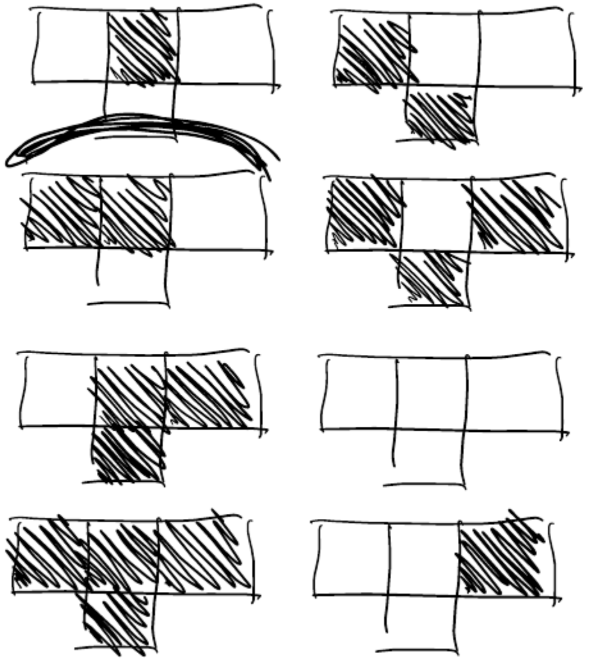
A first Cellular Automaton

“traffic rule”



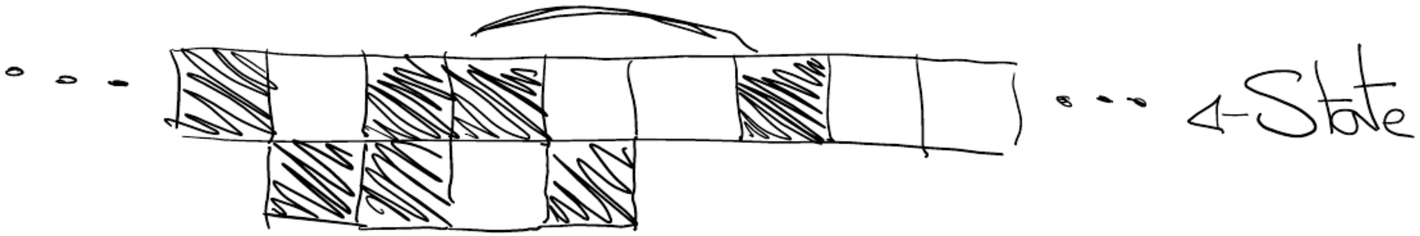
← State

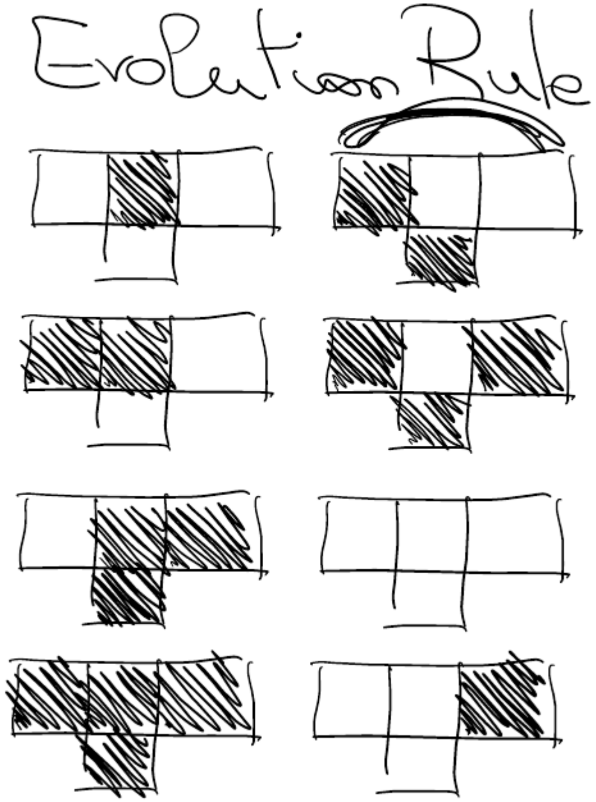
Evolution Rule



A first Cellular Automaton

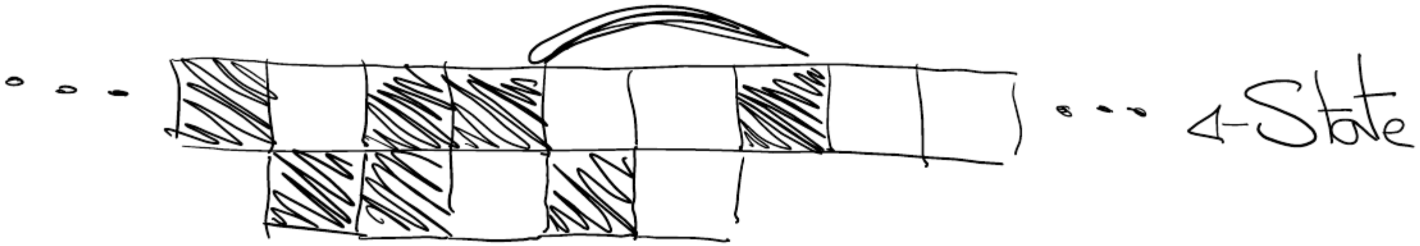
"traffic rule"



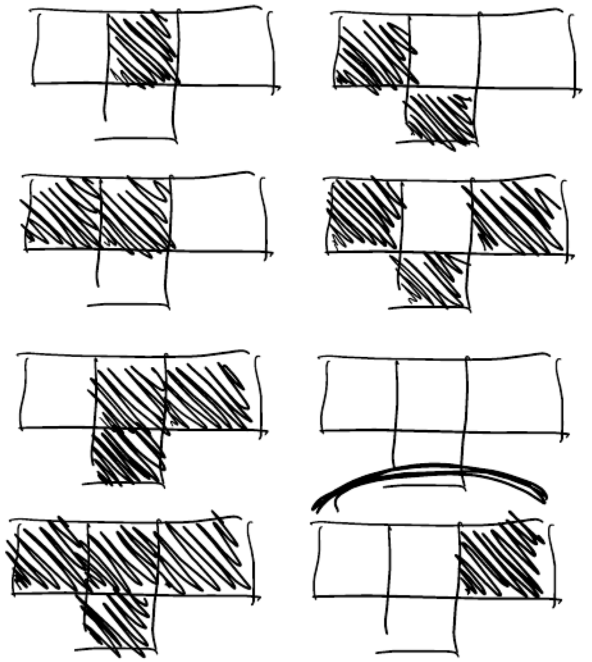


A first Cellular Automaton

"traffic rule"

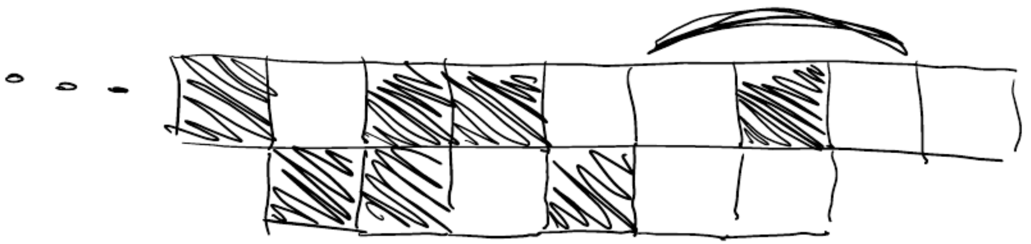


Evolution Rule



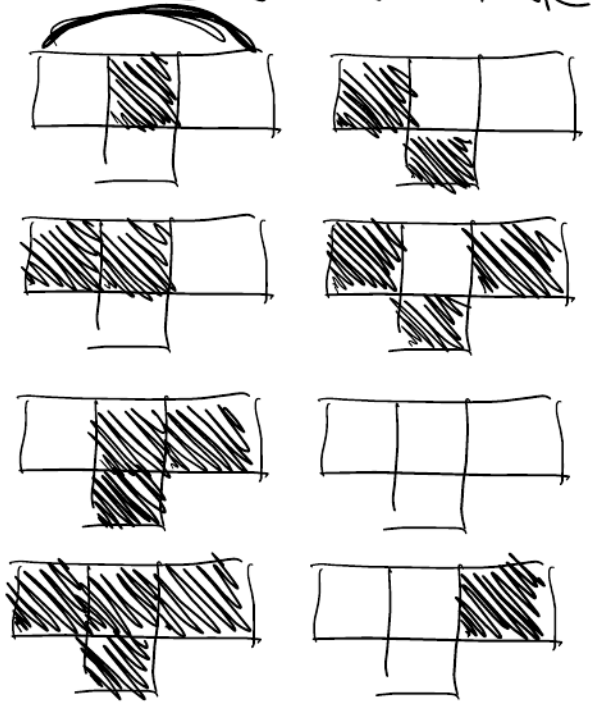
A first Cellular Automaton

“traffic rule”



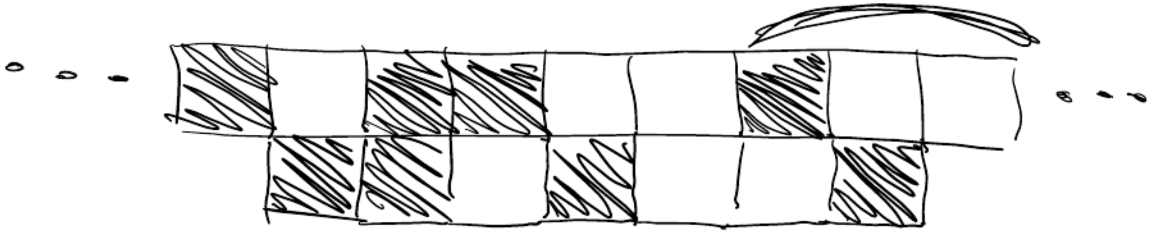
1-State

Evolution Rule



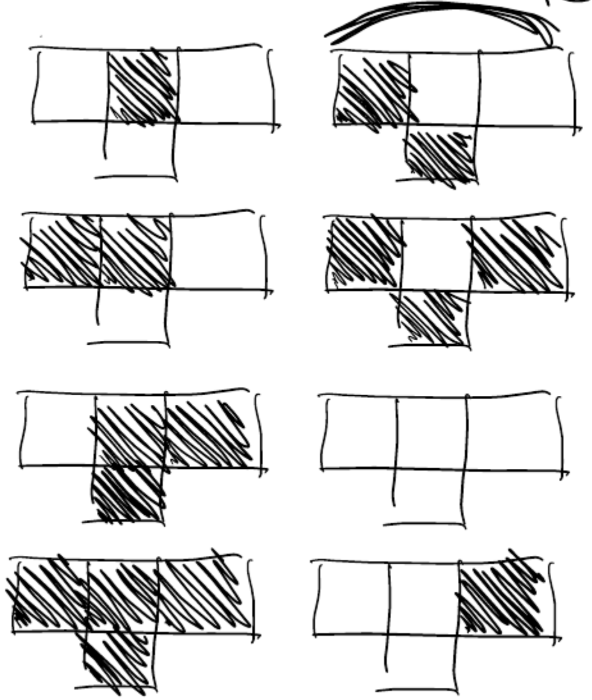
A first Cellular Automaton

“traffic rule”



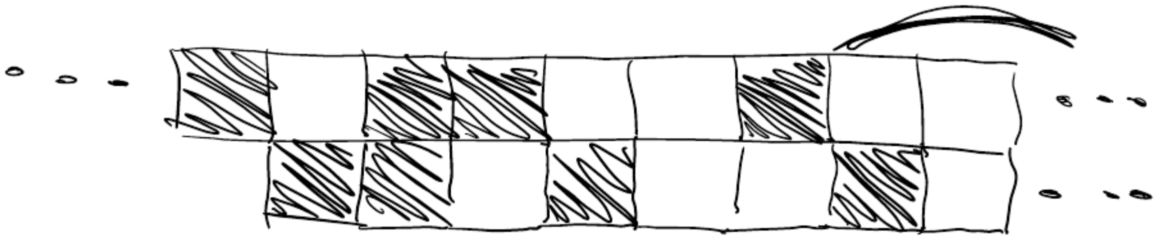
4-State

Evolution Rule



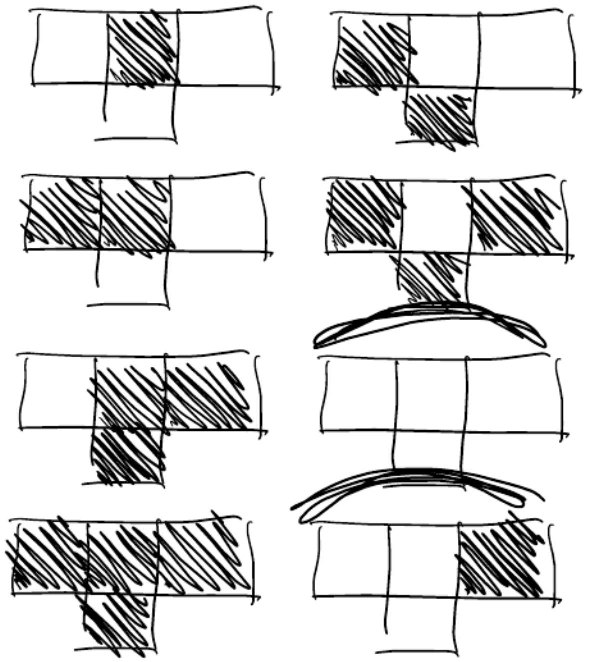
A first Cellular Automaton

"traffic rule"



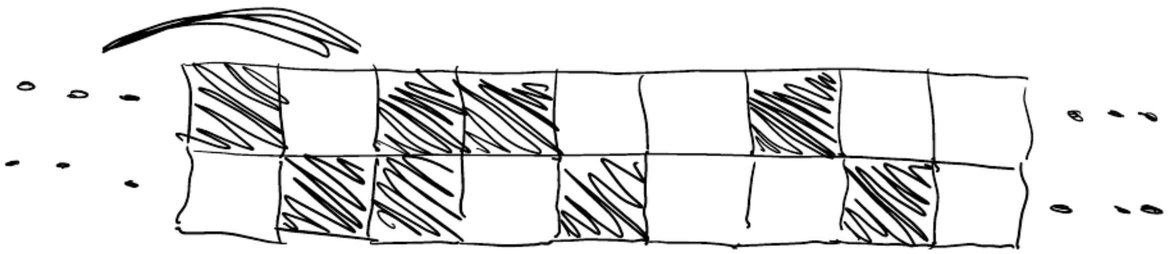
4-State

Evolution Rule



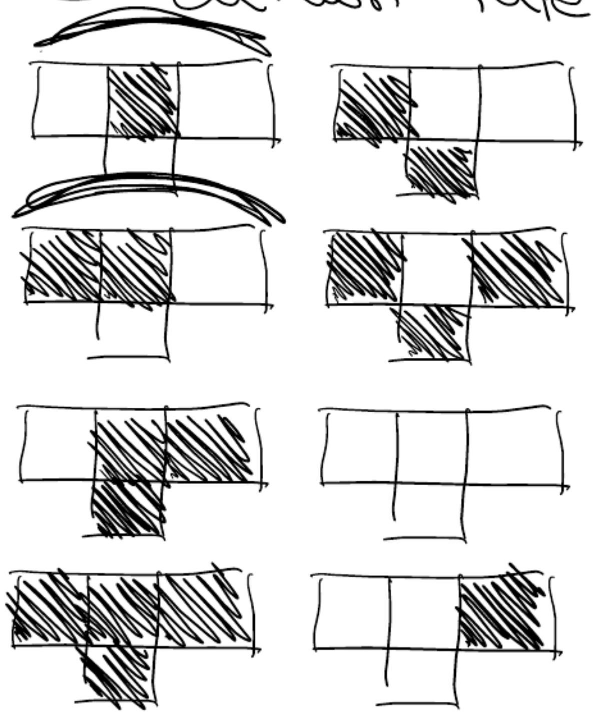
A first Cellular Automaton

"traffic rule"



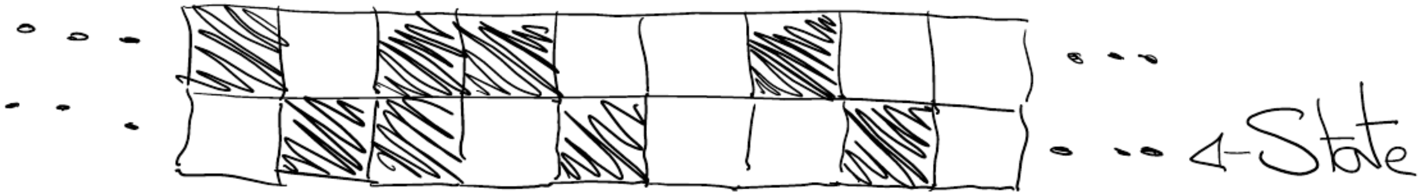
1-State

Evolution Rule

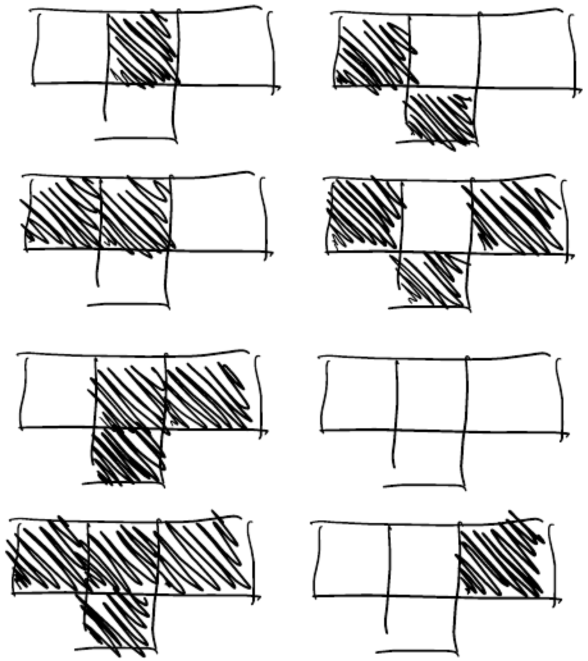


A first Cellular Automaton

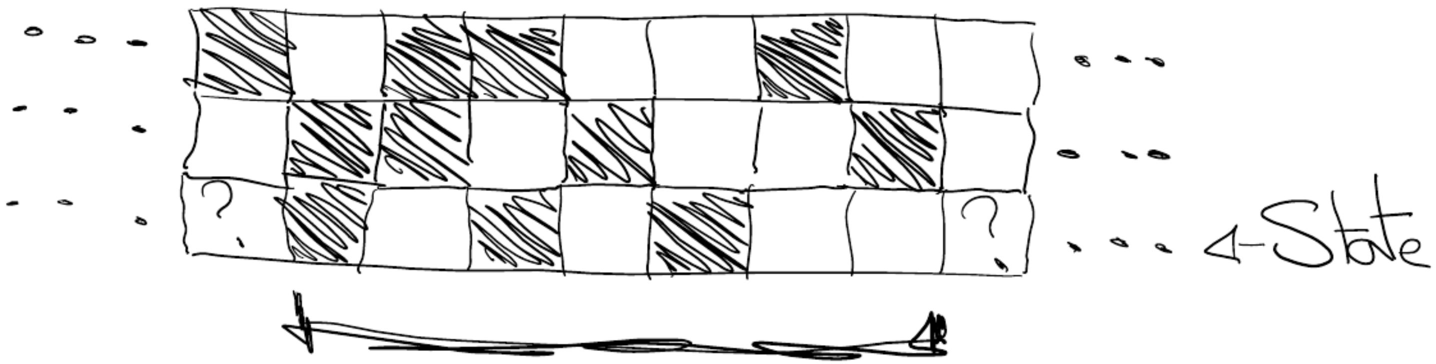
"traffic rule"



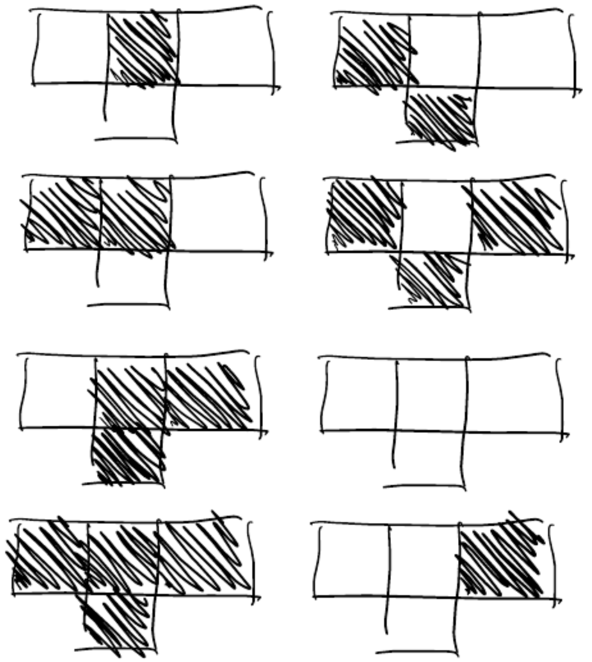
Evolution Rule



A first Cellular Automaton
"traffic rule"

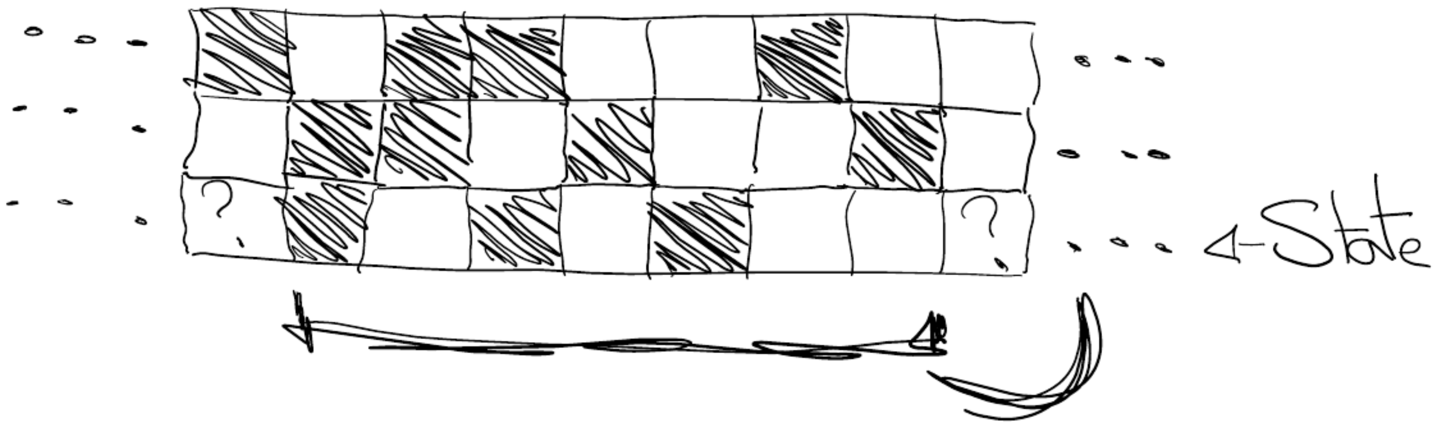


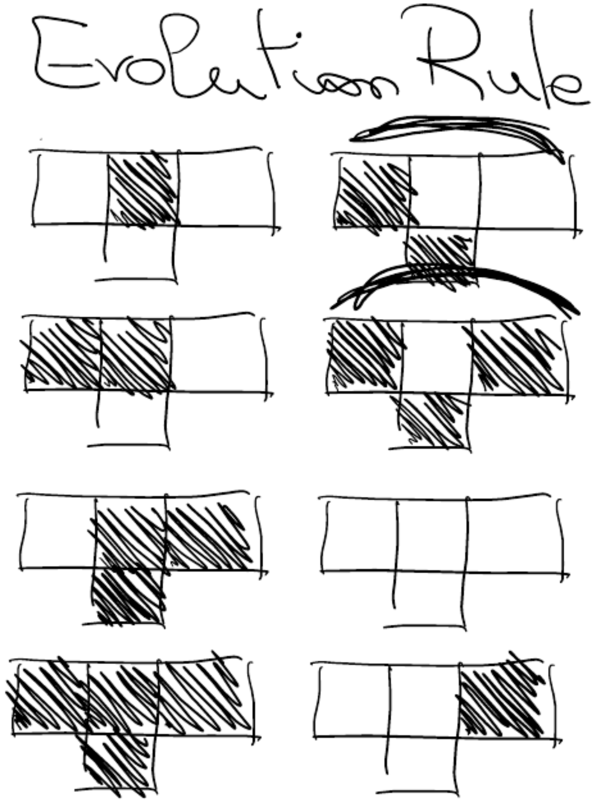
Evolution Rule



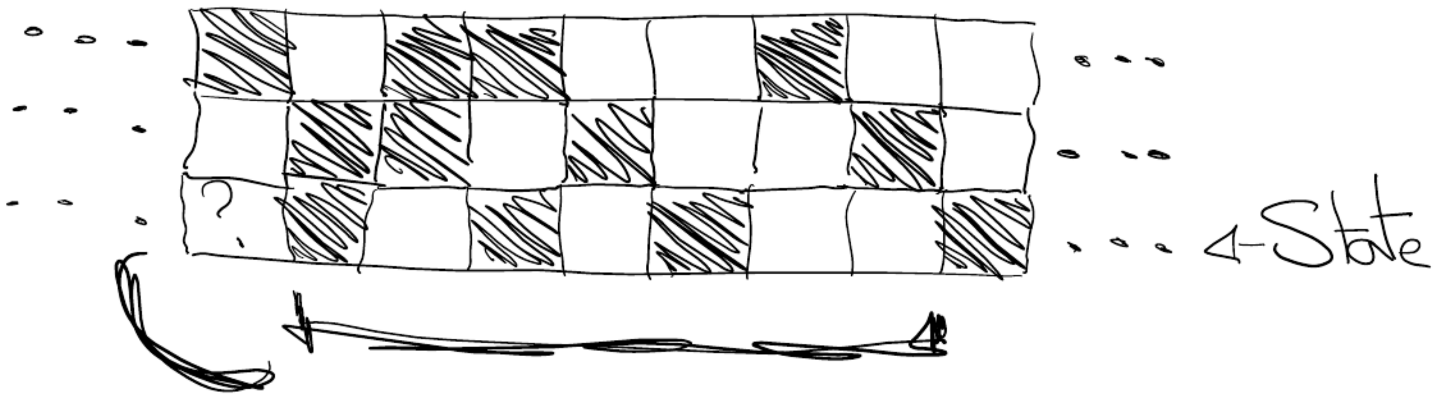
A first Cellular Automaton

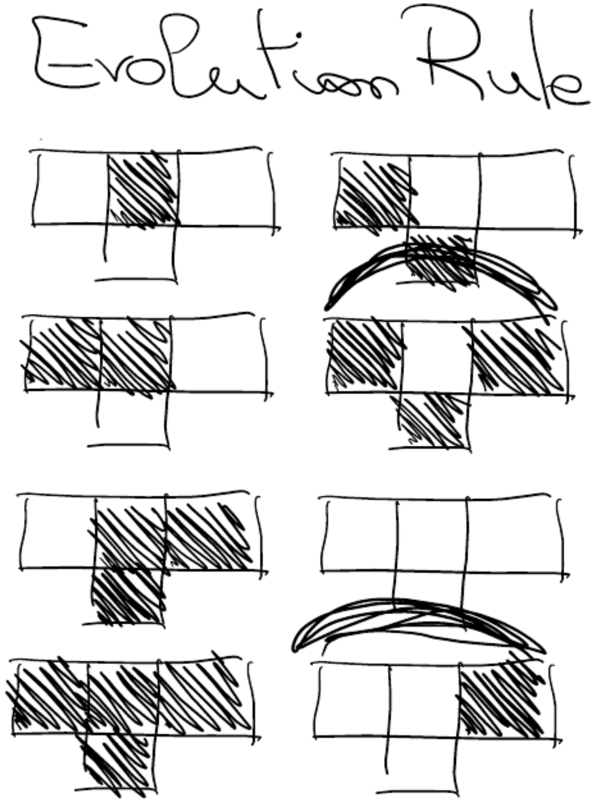
"traffic rule"



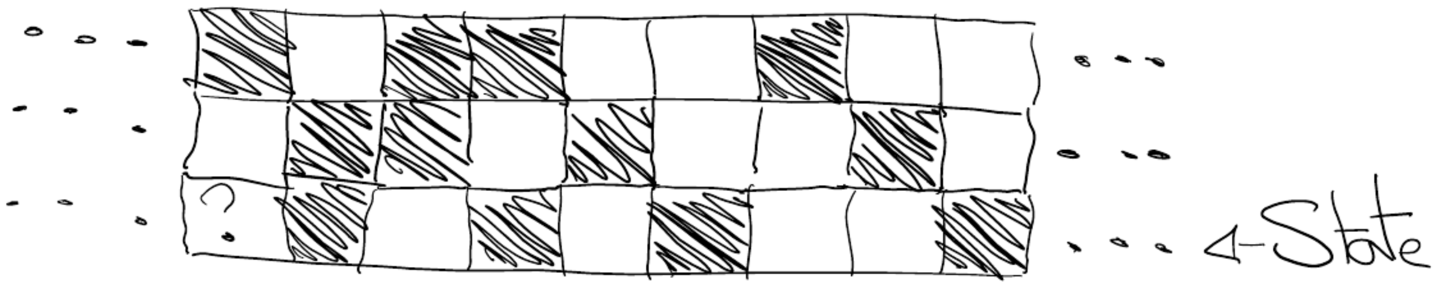


A first Cellular Automaton
"traffic rule"

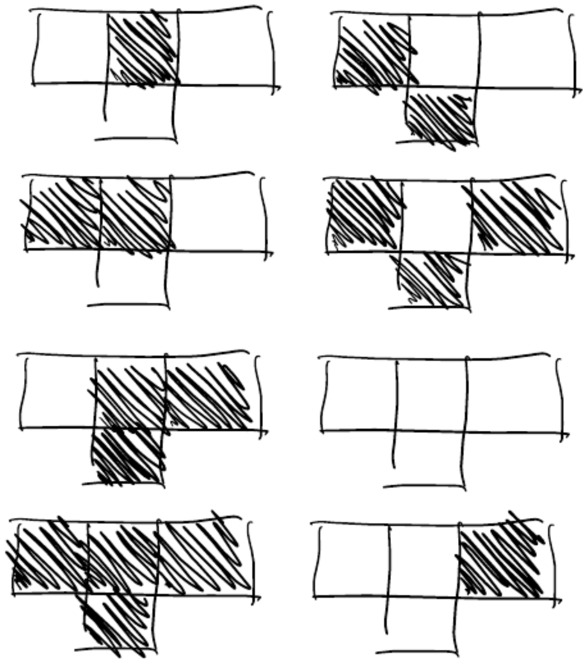




A first Cellular Automaton
"traffic rule"

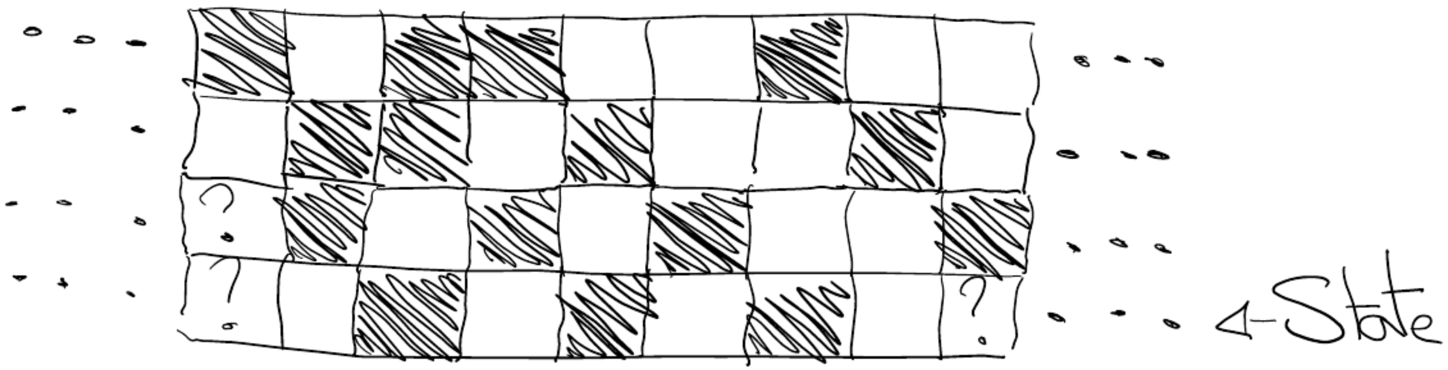


Evolution Rule

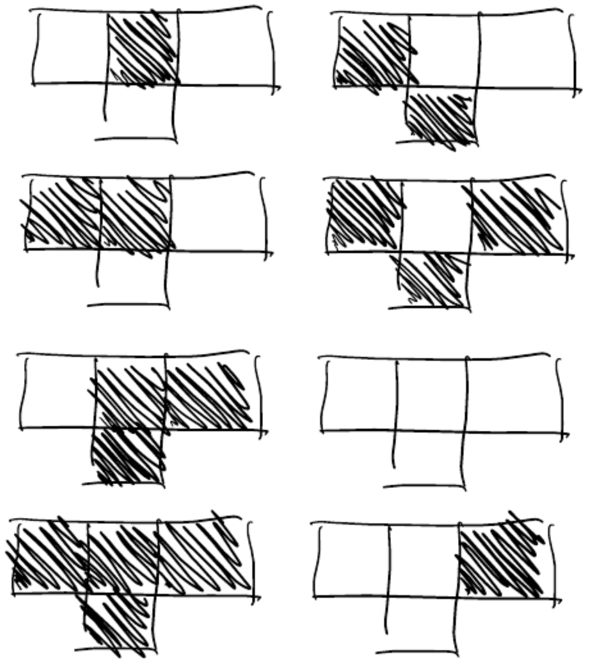


A first Cellular Automaton

"traffic rule"

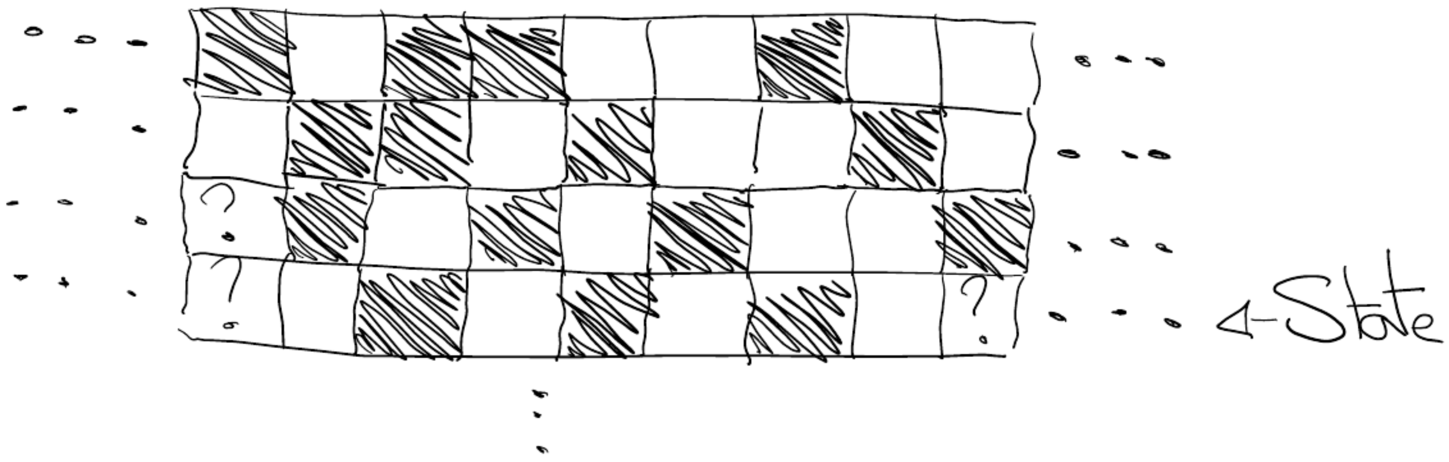


Evolution Rule

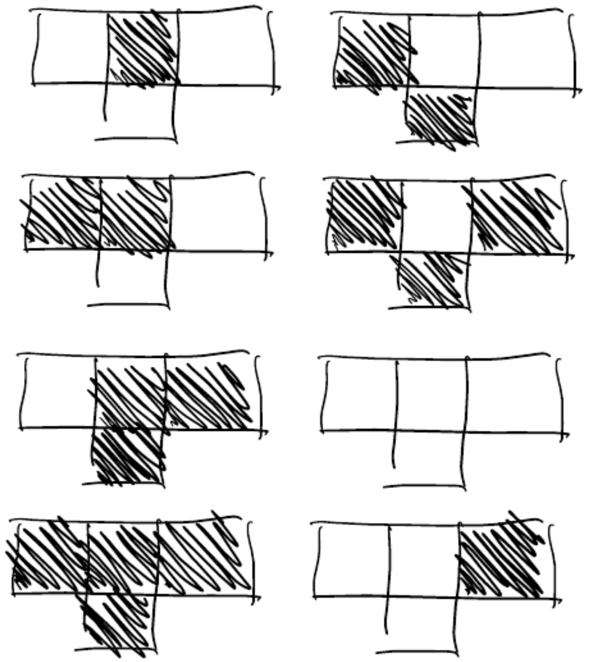


A first Cellular Automaton

"traffic rule"

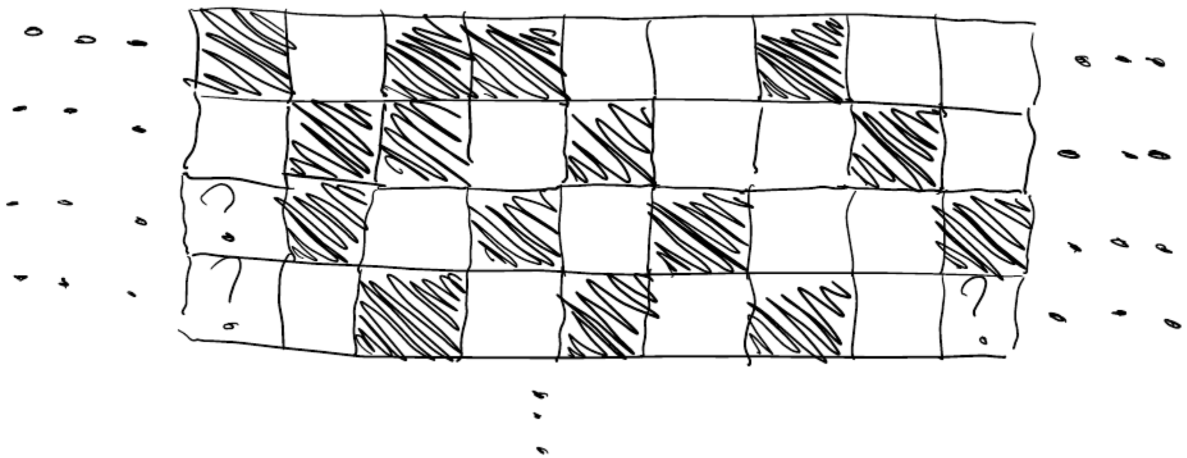


Evolution Rule



A first Cellular Automaton

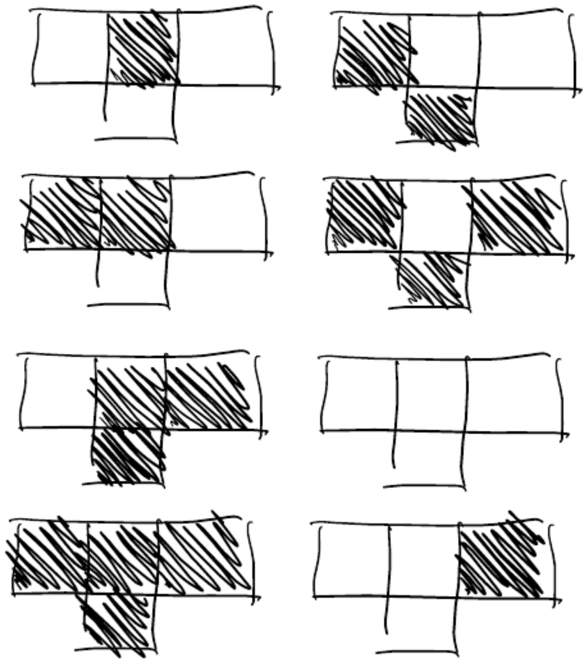
"traffic rule"



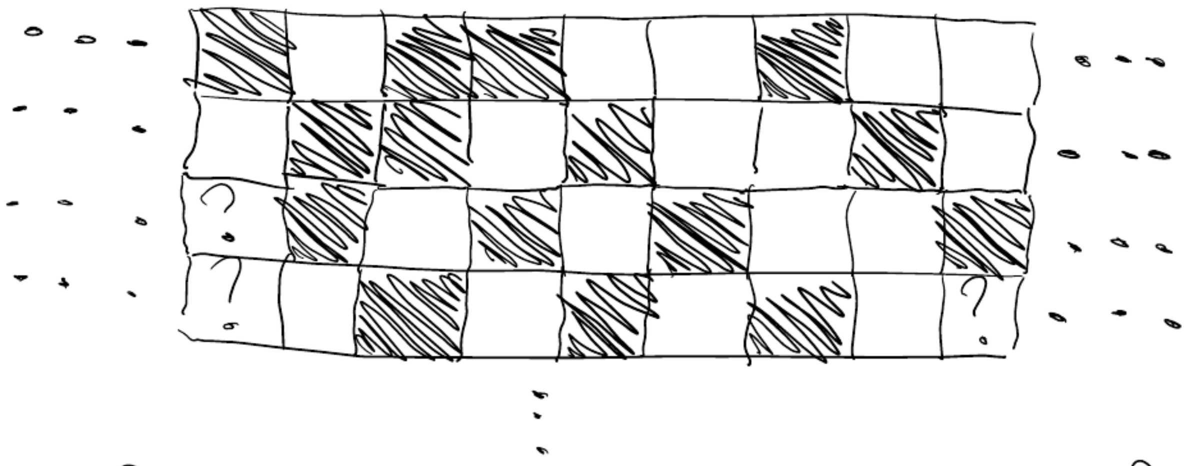
4-State

"informal reasoning, we have infinite conf."

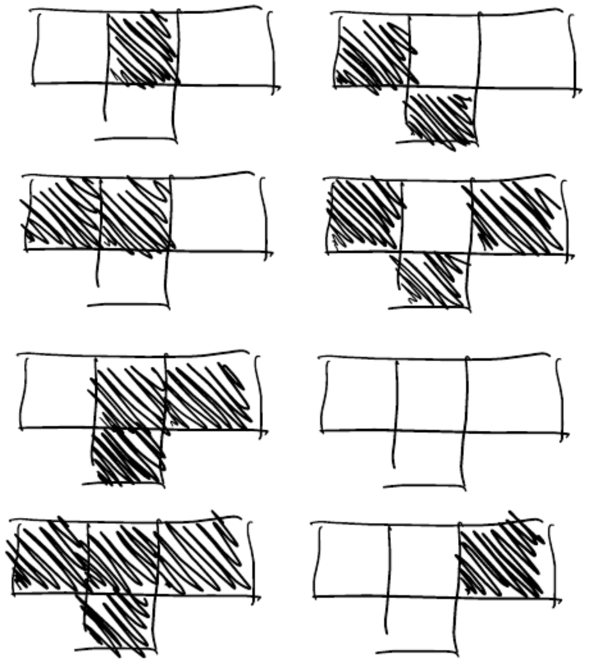
Evolution Rule



A first Cellular Automaton
"traffic rule"



Evolution Rule



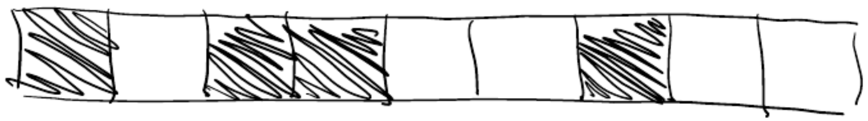
① "informal reasoning, we have infinite conf."

② make it precise!

→ partial configurations

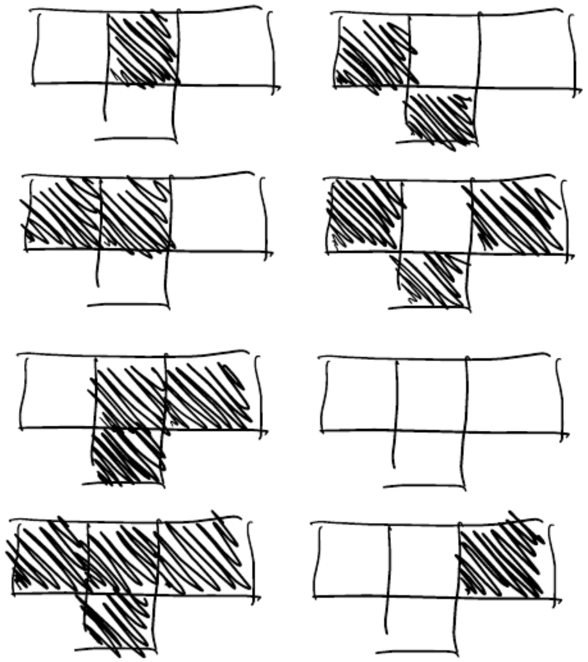
A first Cellular Automaton

"traffic rule"



1-State

Evolution Rule



1. "informal reasoning, we have infinite conf."
2. make it precise!

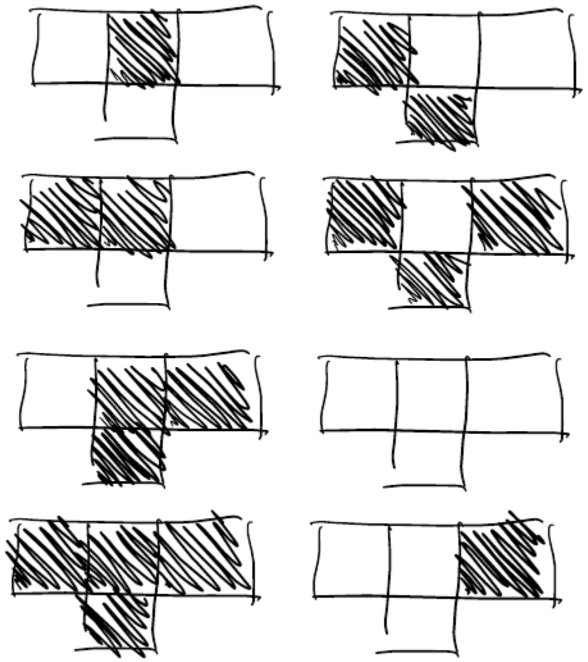
→ partial configurations

A first Cellular Automaton

Most obvious "traffic rule"
and common way

← State

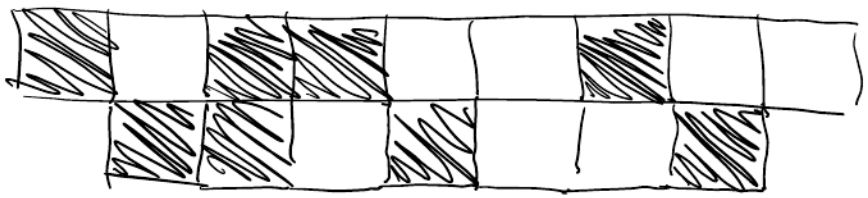
Evolution Rule



① "informal reasoning, we have infinite conf."
② make it precise!
→ partial configurations

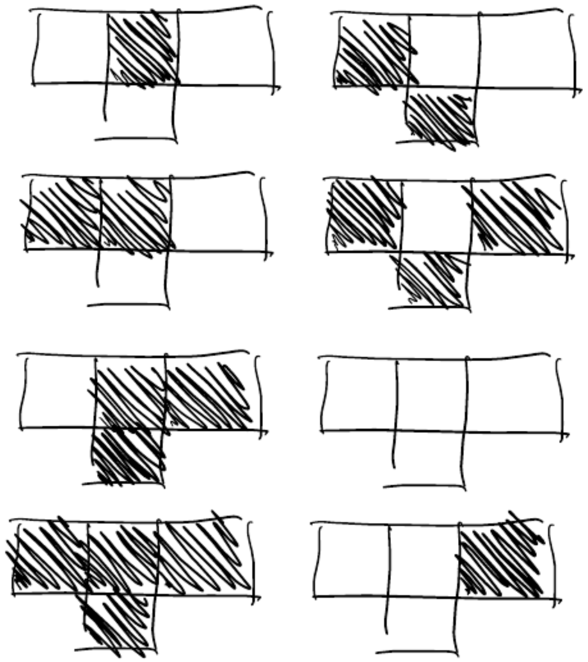
A first Cellular Automaton

Most obvious and common way   "traffic rule"



◁-State

Evolution Rule



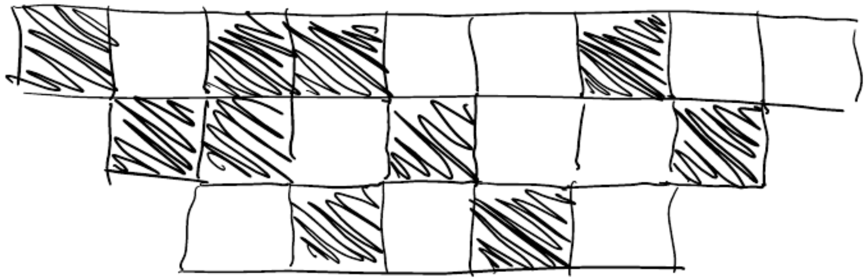
① "informal reasoning, we have infinite conf."

② make it precise!

⟶ partial configurations

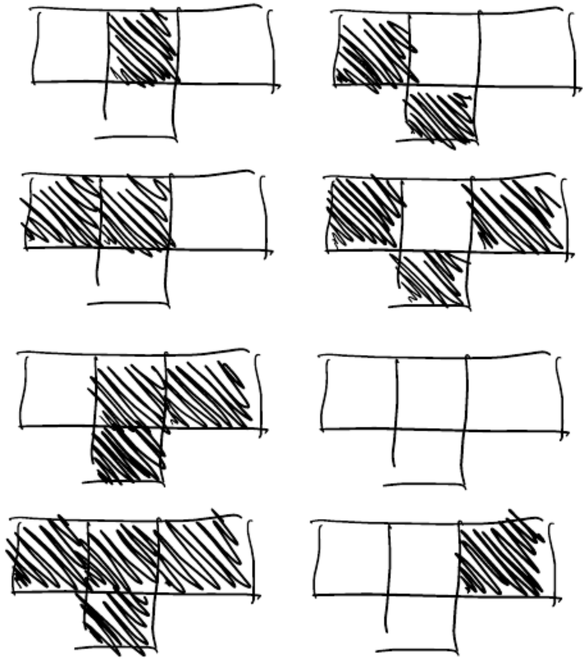
A first Cellular Automaton

Most obvious and common way “traffic rule”



← State

Evolution Rule

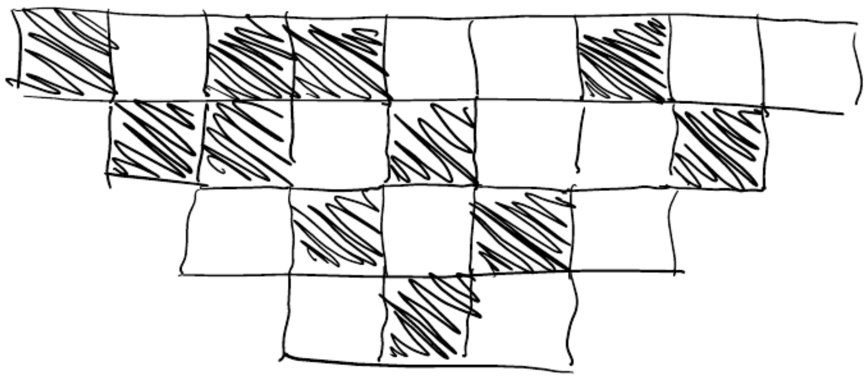


1. “informal reasoning, we have infinite conf.”
2. make it precise!

→ partial configurations

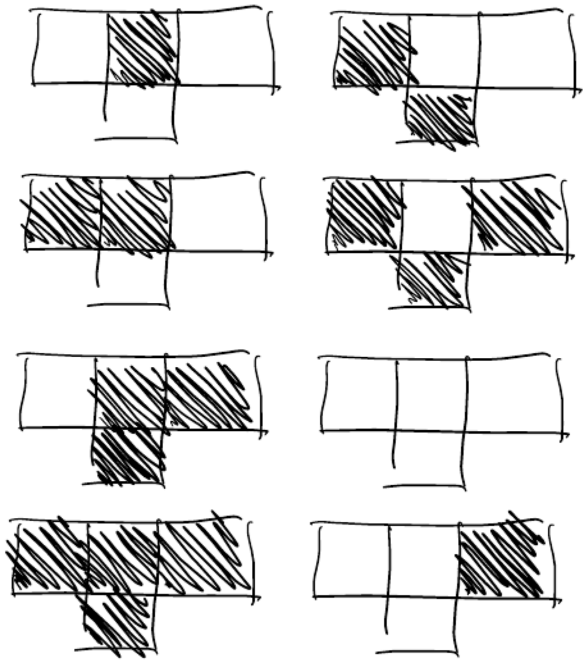
A first Cellular Automaton

Most obvious and common way "traffic rule"



4-State

Evolution Rule



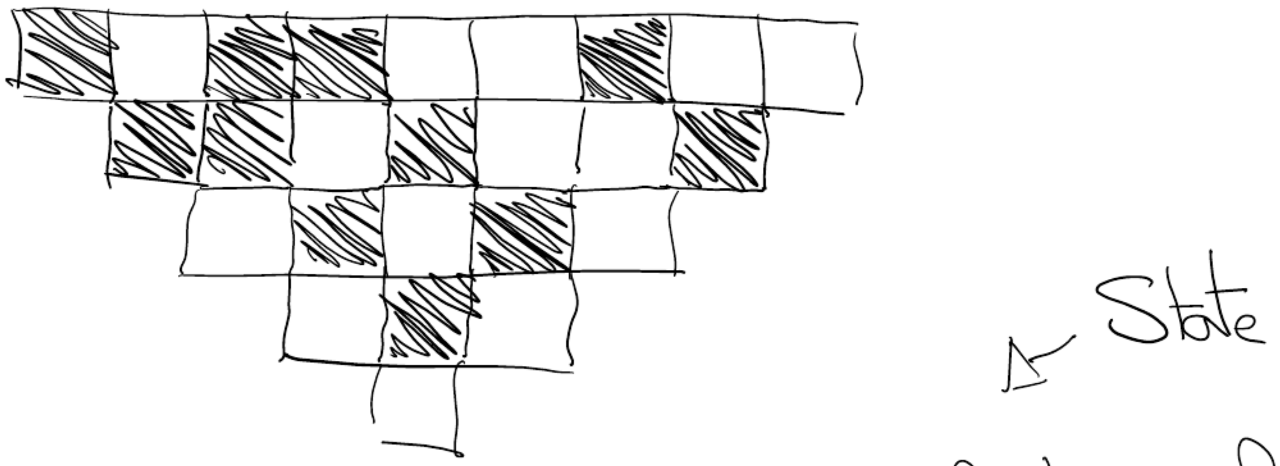
1 "informal reasoning, we have infinite conf."

2 make it precise!

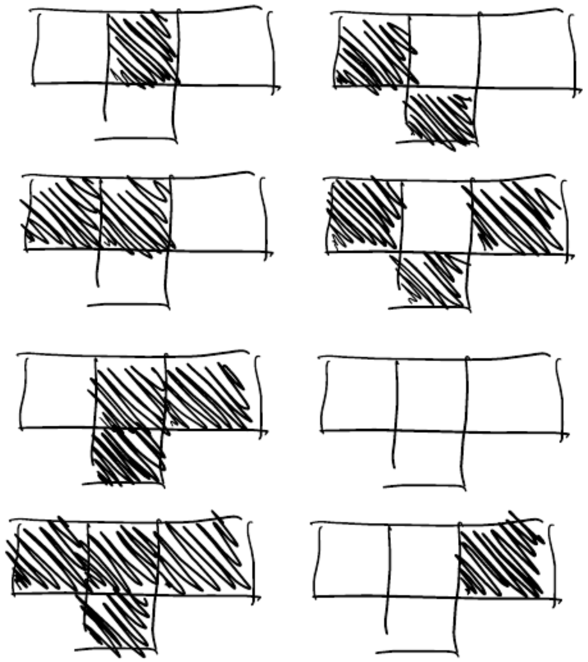
→ partial configurations

A first Cellular Automaton

Most obvious and common way "traffic rule"



Evolution Rule



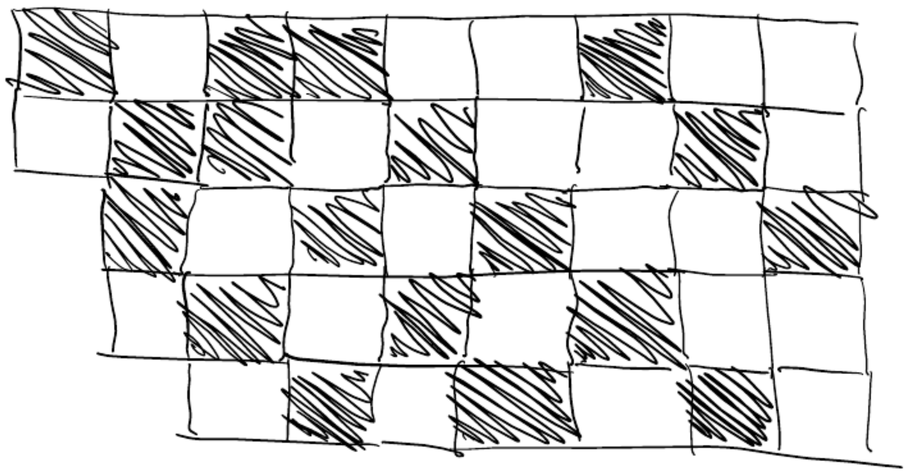
1. "informal reasoning, we have infinite conf."
2. make it precise!

→ partial configurations

A first Cellular Automaton

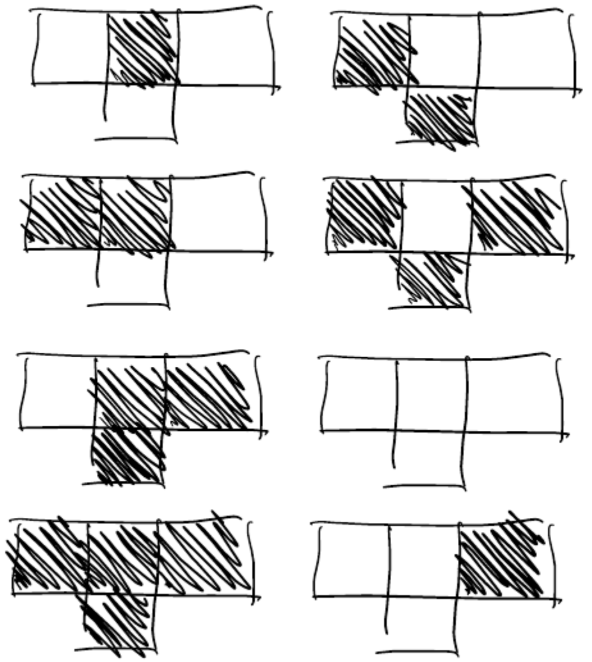
Most ~~obvious and common~~ way complete

“traffic rule”



State

Evolution Rule

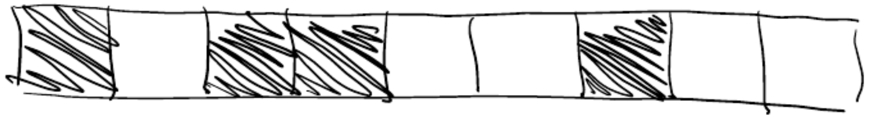


1. “informal reasoning, we have infinite conf.”
2. make it precise !!

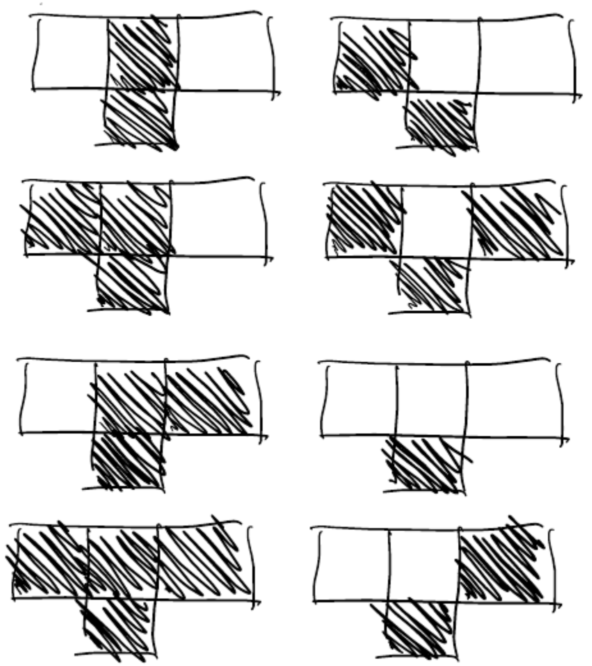
→ partial configurations

A first Cellular Automaton

Most ~~obvious and common~~ way "~~traffic~~ rule"
complete constant



Evolution Rule

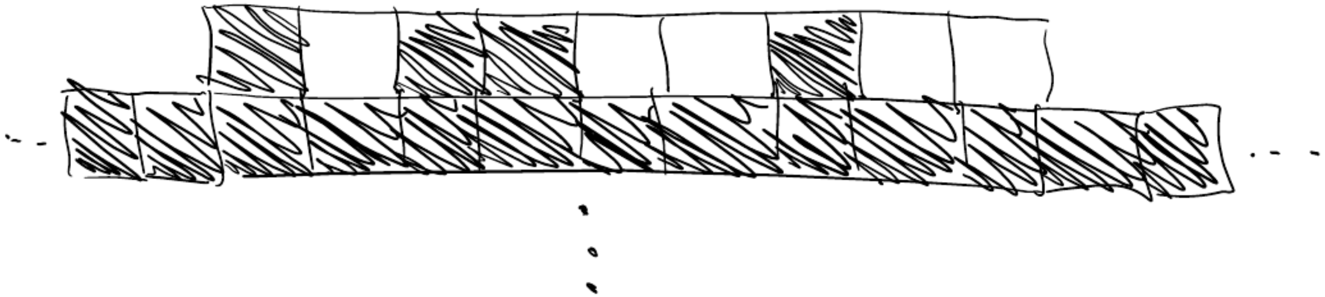


1. "informal reasoning, we have infinite conf."
2. make it precise!!

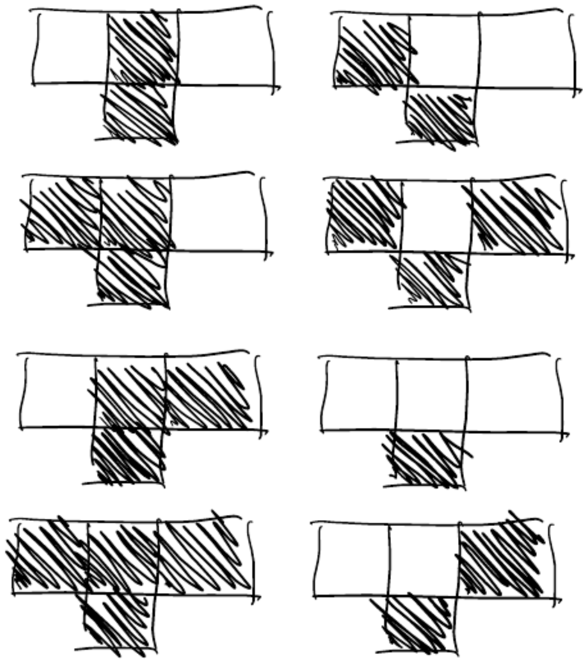
→ partial configurations

A first Cellular Automaton

Most ~~obvious and common~~ complete way “~~traffic~~ constant rule”



Evolution Rule



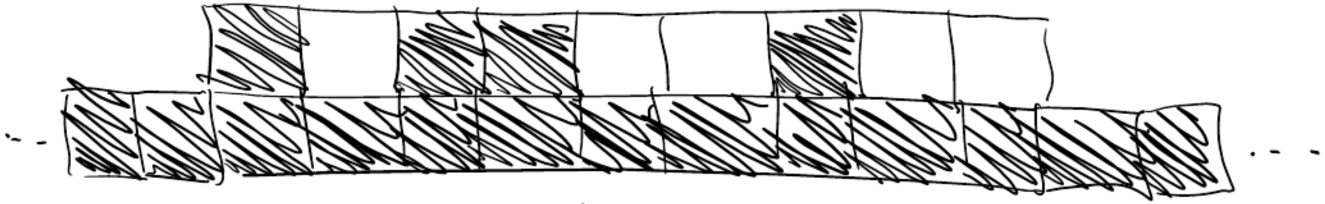
① “informal reasoning, we have infinite conf.”

② make it precise!

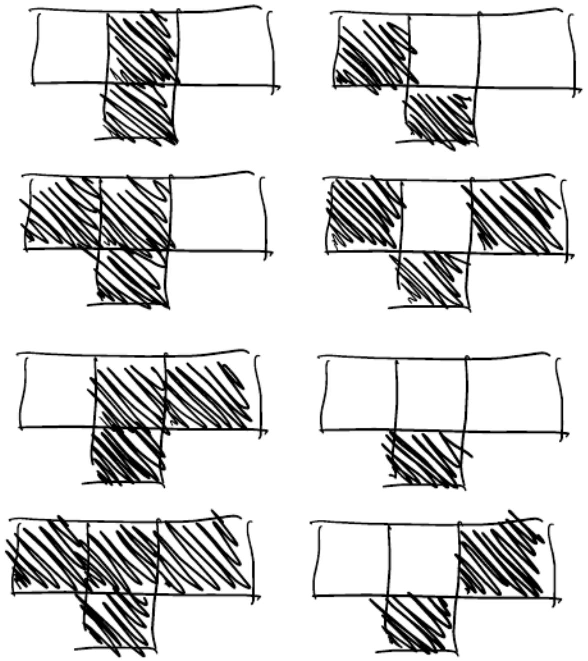
→ partial configurations

A first Cellular Automaton

Most ~~obvious~~ ~~and common~~ complete way "~~traffic~~ constant rule"



Evolution Rule



Two optimal ways

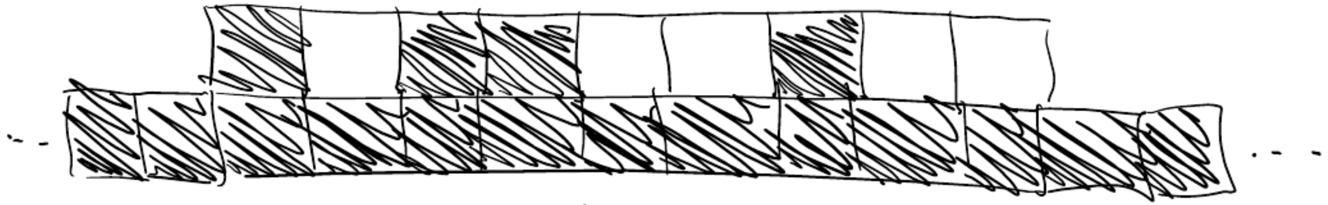
1. "informal reasoning, we have infinite conf."
2. make it precise!

→ partial configurations

A first Cellular Automaton

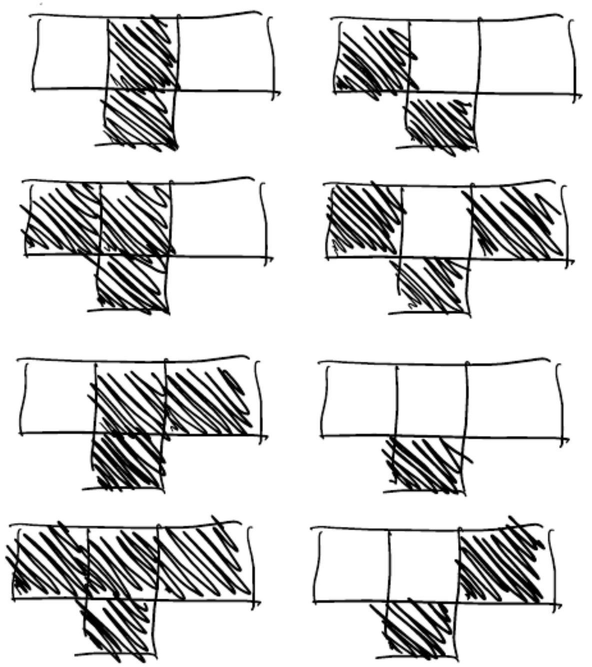
Most ~~obvious~~ ~~and common~~ complete way

“~~traffic~~ constant rule”



Yes, we Kan : Two optimal ways

Evolution Rule



1. “informal reasoning, we have infinite conf.”
2. make it precise!

→ partial configurations

Firsts Cellular Automata

Firsts Cellular Automata

- Q: set of states
- $N \subsetneq \mathbb{Z}$: finite set of neighbor index
- $\delta : Q^N \to Q$: local transition function

Firsts Cellular Automata

$\{\square, \blacksquare\}$

- Q : set of states
- $N \subsetneq \mathbb{Z}$: finite set of neighbor index
- $\delta : Q^N \to Q$: local transition function

Firsts Cellular Automata

- Q: set of states $\{\square, \blacksquare\}$
- $N \subsetneq \mathbb{Z}$: finite set of neighbor index $\{-1, 0, 1\}$
- $\delta : Q^N \to Q$: local transition function

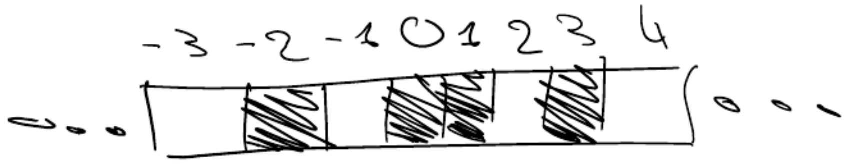
Firsts Cellular Automata

- Q : set of states $\{\square, \blacksquare\}$
- $N \subsetneq \mathbb{Z}$: finite set of neighbor index $\{-1, 0, 1\}$
- $\delta : Q^N \to Q$: local transition function "traffic rule"

Firsts Cellular Automata

- Q : set of states $\quad \{\square, \blacksquare\}$
- $N \subsetneq \mathbb{Z}$: finite set of neighbor index $\quad \{-1, 0, 1\}$
- $\delta : Q^N \to Q$: local transition function "traffic rule"

configurations : $c \in Q^{\mathbb{Z}}$



Firsts Cellular Automata

- Q: set of states $\{\square, \blacksquare\}$
- $N \subsetneq \mathbb{Z}$: finite set of neighbor index $\{-1, 0, 1\}$
- $\delta : Q^N \to Q$: local transition function "traffic rule"

configurations: $c \in Q^{\mathbb{Z}}$

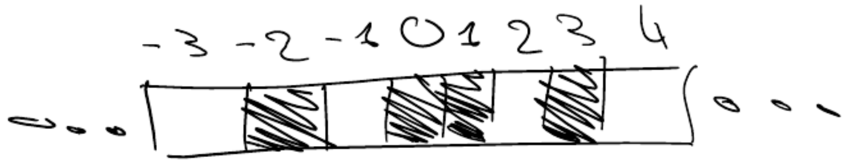
global transition function: $\Delta : Q^{\mathbb{Z}} \longrightarrow Q^{\mathbb{Z}}$

$c \mapsto$

Firsts Cellular Automata

- Q : set of states $\{\square, \blacksquare\}$
- $N \subsetneq \mathbb{Z}$: finite set of neighbor index $\{-1, 0, 1\}$
- $\delta : Q^N \to Q$: local transition function "traffic rule"

configurations : $c \in Q^{\mathbb{Z}}$



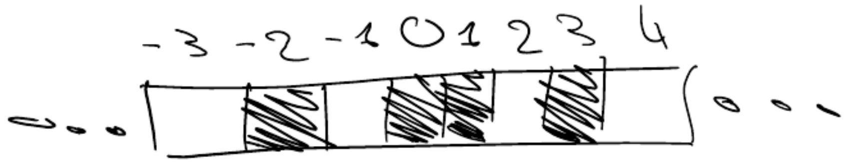
global transition function : $\Delta : Q^{\mathbb{Z}} \longrightarrow Q^{\mathbb{Z}}$

$c \mapsto (p \mapsto \qquad)$

Firsts Cellular Automata

- Q: set of states $\{\square, \blacksquare\}$
- $N \subsetneq \mathbb{Z}$: finite set of neighbor index $\{-1, 0, 1\}$
- $\delta : Q^N \to Q$: local transition function "traffic rule"

configurations: $c \in Q^{\mathbb{Z}}$



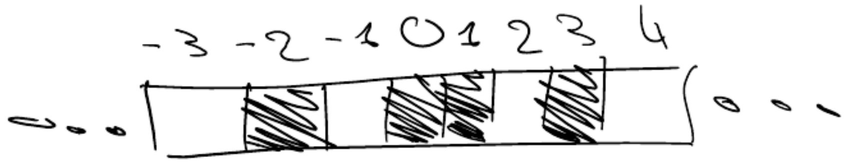
global transition function: $\Delta : Q^{\mathbb{Z}} \longrightarrow Q^{\mathbb{Z}}$

$c \mapsto (p \mapsto (n \mapsto c(p+n)))$

Firsts Cellular Automata

- Q : set of states $\{\square, \blacksquare\}$
- $N \subsetneq \mathbb{Z}$: finite set of neighbor index $\{-1, 0, 1\}$
- $\delta : Q^N \to Q$: local transition function "traffic rule"

configurations : $c \in Q^{\mathbb{Z}}$

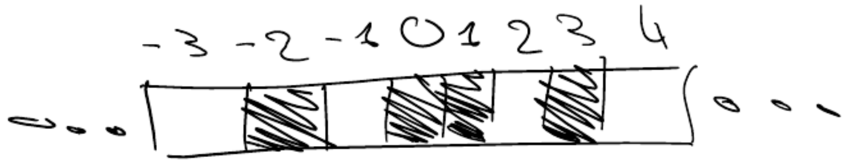


global transition function : $\Delta : Q^{\mathbb{Z}} \longrightarrow Q^{\mathbb{Z}}$

$$c \mapsto (p \mapsto \delta(n \mapsto c(p+n)))$$

Firsts Cellular Automata

configurations : $c \in Q^{\mathbb{Z}}$



global transition function : $\Delta : Q^{\mathbb{Z}} \longrightarrow Q^{\mathbb{Z}}$

$$c \mapsto (p \mapsto \delta(n \mapsto c(p+n)))$$

Firsts Cellular Automata
Extension to Partial Configurations

configurations: $c \in Q^{\mathbb{Z}}$

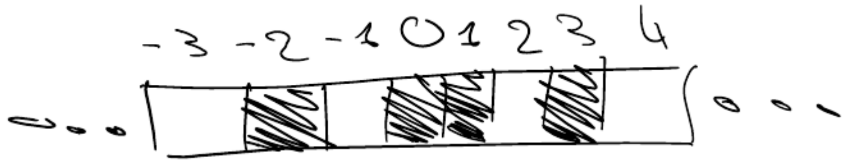
global transition function: $\Delta : Q^{\mathbb{Z}} \longrightarrow Q^{\mathbb{Z}}$

$$c \mapsto \left(p \mapsto \delta(n \mapsto c(p+n))\right)$$

Firsts Cellular Automata
Extension to Partial Configurations

$$\underline{\Delta} = \mathrm{Lan}_{\underline{I}} \overleftrightarrow{\delta}$$

configurations : $c \in Q^{\mathbb{Z}}$



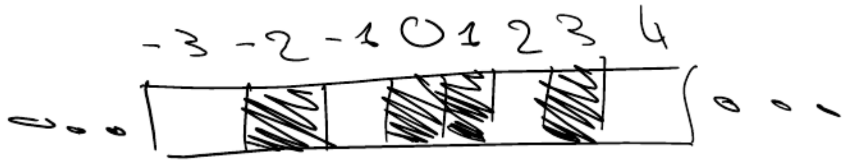
global transition function : $\Delta : Q^{\mathbb{Z}} \longrightarrow Q^{\mathbb{Z}}$

$$c \mapsto (p \mapsto \delta(n \mapsto c(p+n)))$$

Firsts Cellular Automata
Extension to Partial Configurations

$$\underline{\Delta} = \mathrm{Lan}_{I} \overleftrightarrow{\delta} \qquad\qquad \overline{\Delta} = \mathrm{Ran}_{J} \Delta$$

configurations : $c \in Q^{\mathbb{Z}}$



global transition function : $\Delta : Q^{\mathbb{Z}} \longrightarrow Q^{\mathbb{Z}}$

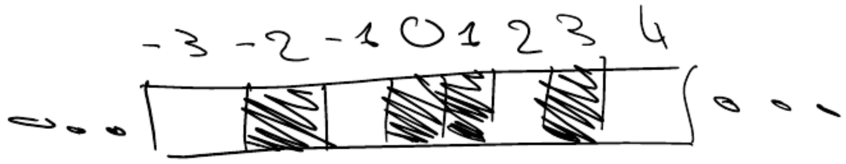
$$c \mapsto (p \mapsto \delta(n \mapsto c(p+n)))$$

Firsts Cellular Automata
Extension to Partial Configurations

$$\underline{\Delta} = \mathrm{Lan}_I \overleftrightarrow{\delta} \qquad \overline{\Delta} = \mathrm{Ran}_J \Delta$$

$$\Theta = id$$

configurations: $c \in Q^{\mathbb{Z}}$



global transition function: $\Delta : Q^{\mathbb{Z}} \longrightarrow Q^{\mathbb{Z}}$

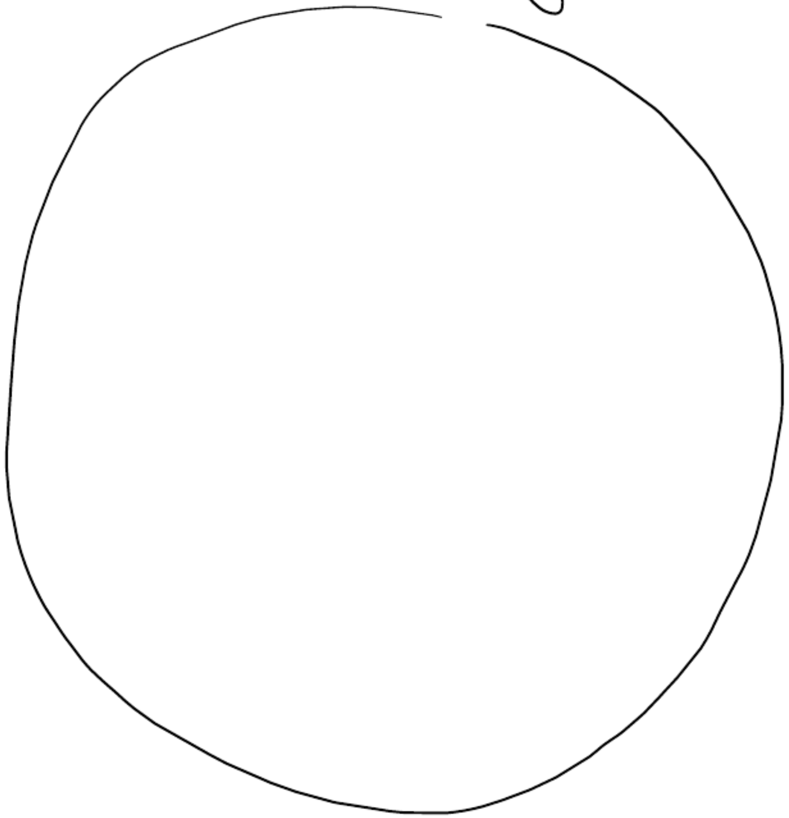
$$c \mapsto (p \mapsto \delta(n \mapsto c(p+n)))$$

Firsts Cellular Automata Extension to Partial Configurations

Firsts Cellular Automata

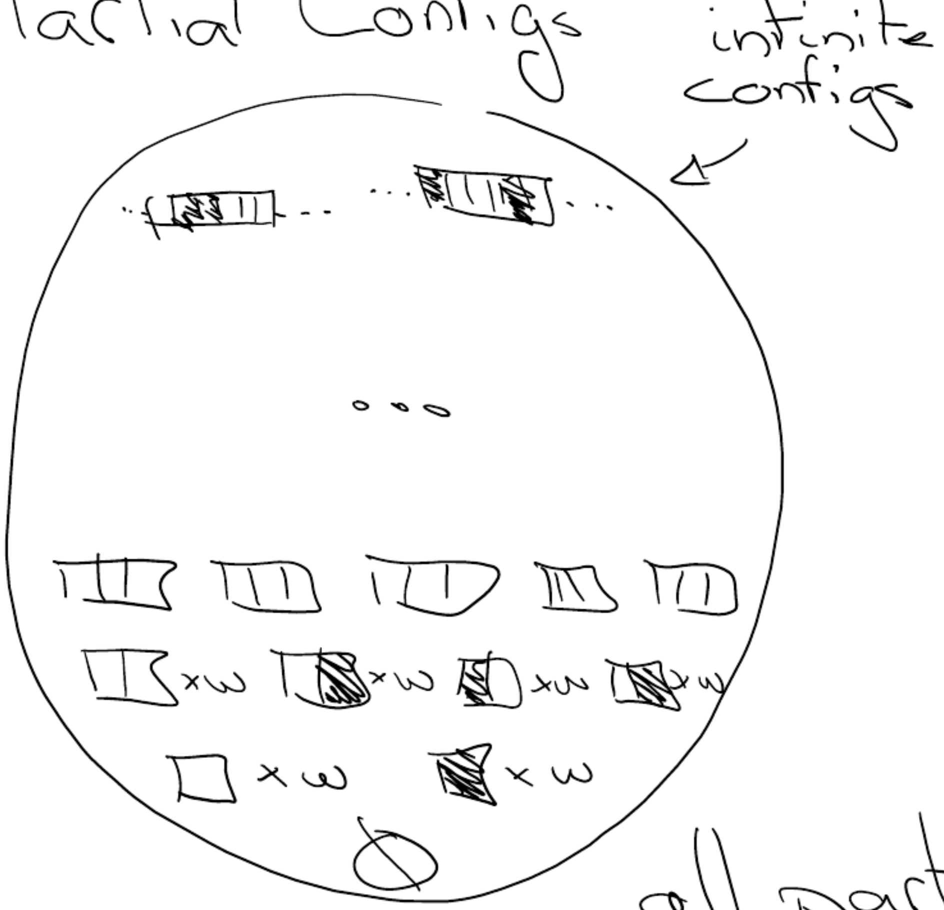
Extension to Partial Configurations

Partial Configs



Firsts Cellular Automata Extension to Partial Configurations

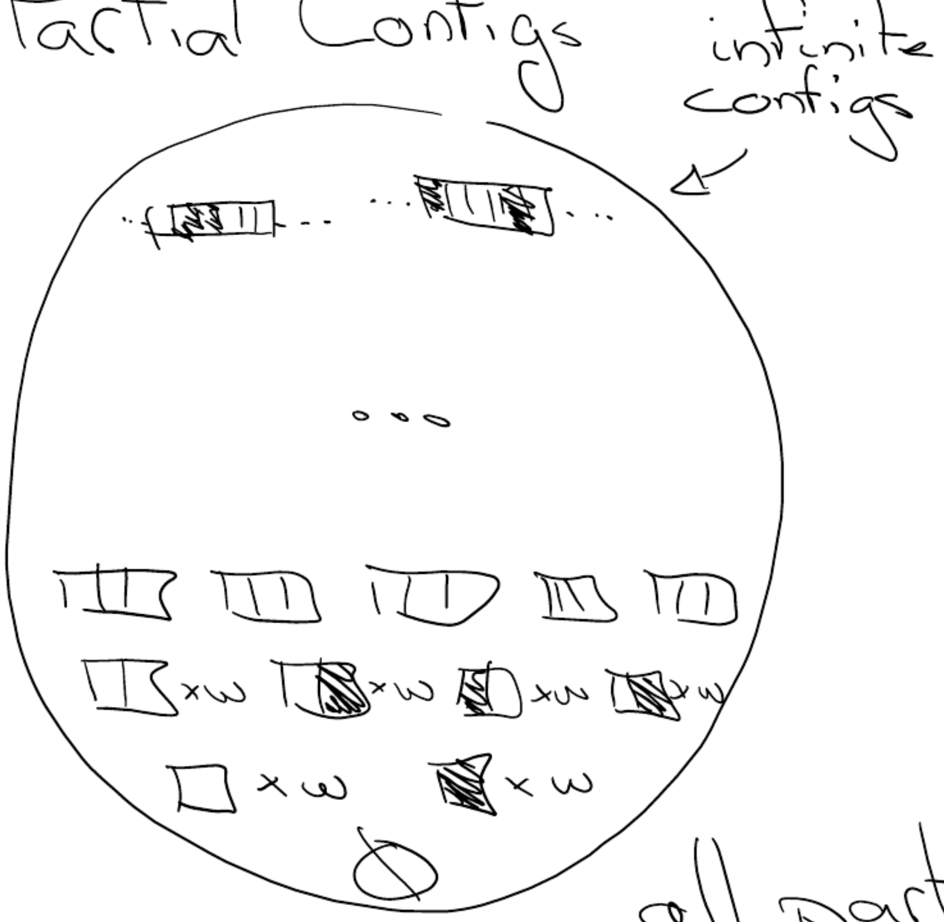
Partial Configs



all partial funtions $\mathbb{Z}_r \to Q$

Firsts Cellular Automata
Extension to Partial Configurations

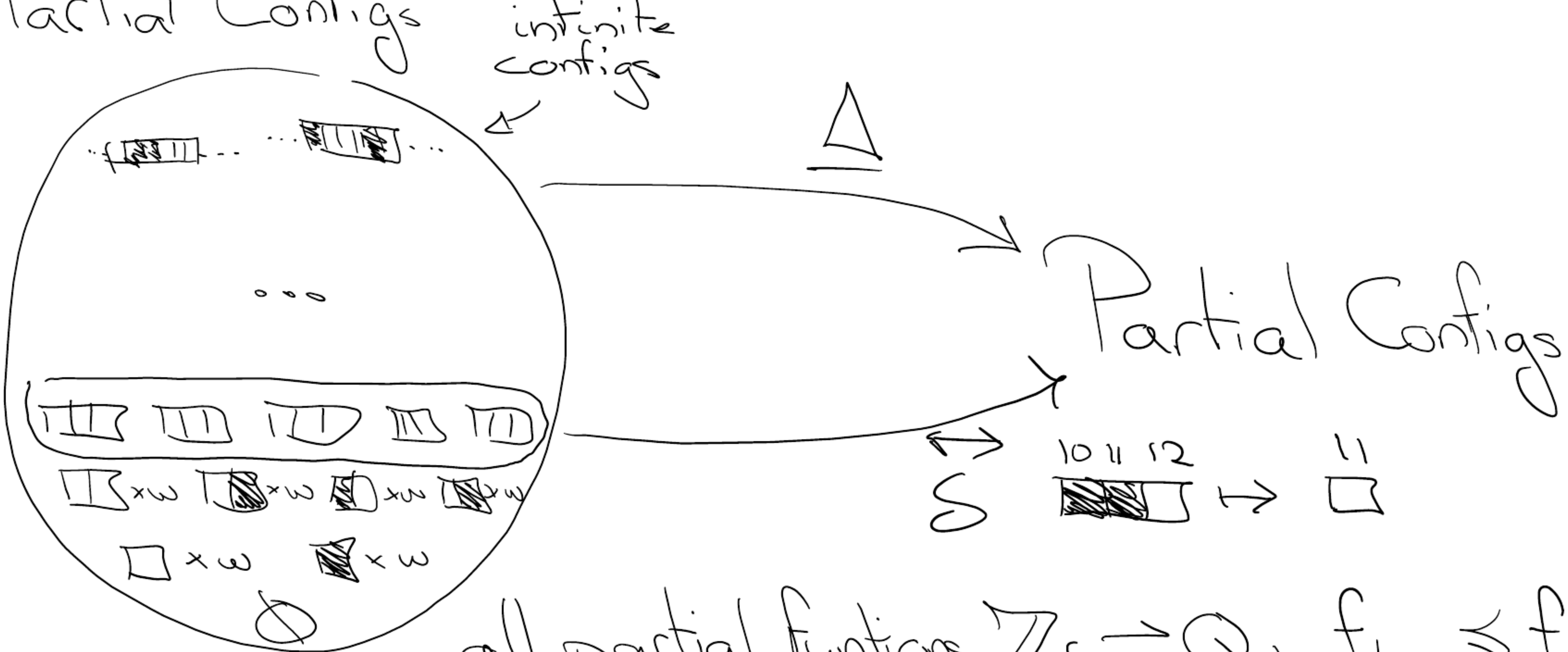
Partial Configs



all partial funtions $\mathbb{Z}_r \to Q$, $f|_S \leq f$

Firsts Cellular Automata Extension to Partial Configurations

Partial Configs



all partial funtions $\mathbb{Z}_r \to Q$, $f|_S \leq f$

Firsts Cellular Automata
Extension to Partial Configurations

Partial Configs

Partial Configs

$F \preccurlyeq G$

$\forall c,\ F(c) \preccurlyeq G(c)$

$\longleftrightarrow$

S 10 11 12 $\mapsto$ 11

all partial funtions $\mathbb{Z}_r \to Q$, $f_{|S} \preccurlyeq f$

Firsts Cellular Automata
Extension to Partial Configurations

Partial Configs

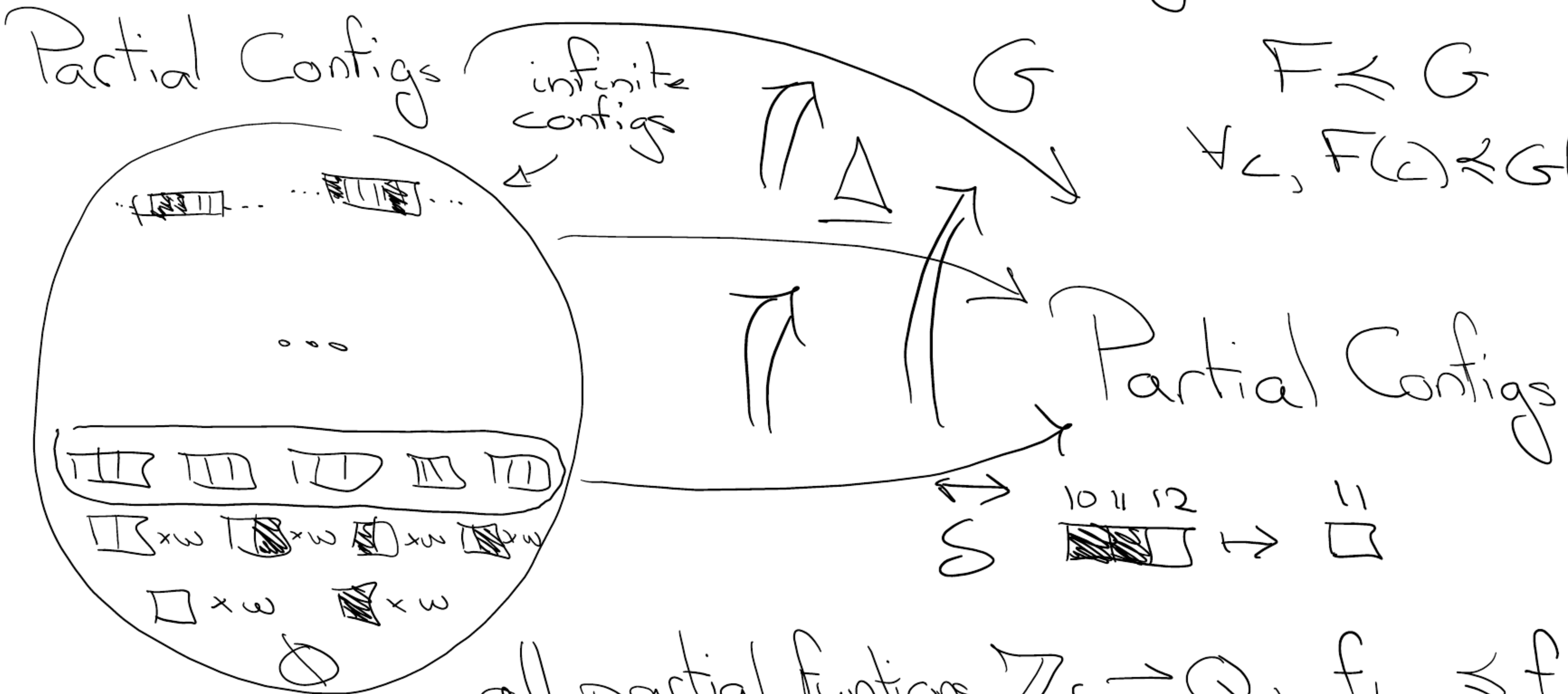
$F \leq G$

$\forall c,\ F(c) \leq G(c)$

Partial Configs

all partial funtions $\mathbb{Z}_r \to Q$, $f_{|S} \leq f$

Firsts Cellular Automata
Extension to Partial Configurations



$F \leq G$

$\forall c,\ F(c) \leq G(c)$

all partial funtions $\mathbb{Z}_S \to Q$, $f_{|S} \leq f$

Firsts Cellular Automata
Extension to Partial Configurations

$\longleftrightarrow \delta$

Partial Configs

infinite configs

$\mathrm{Lan}_I \, \underline{\Delta}$

$F \leq G$

$\forall c, F(c) \leq G(c)$

Partial Configs

$\longleftrightarrow S$

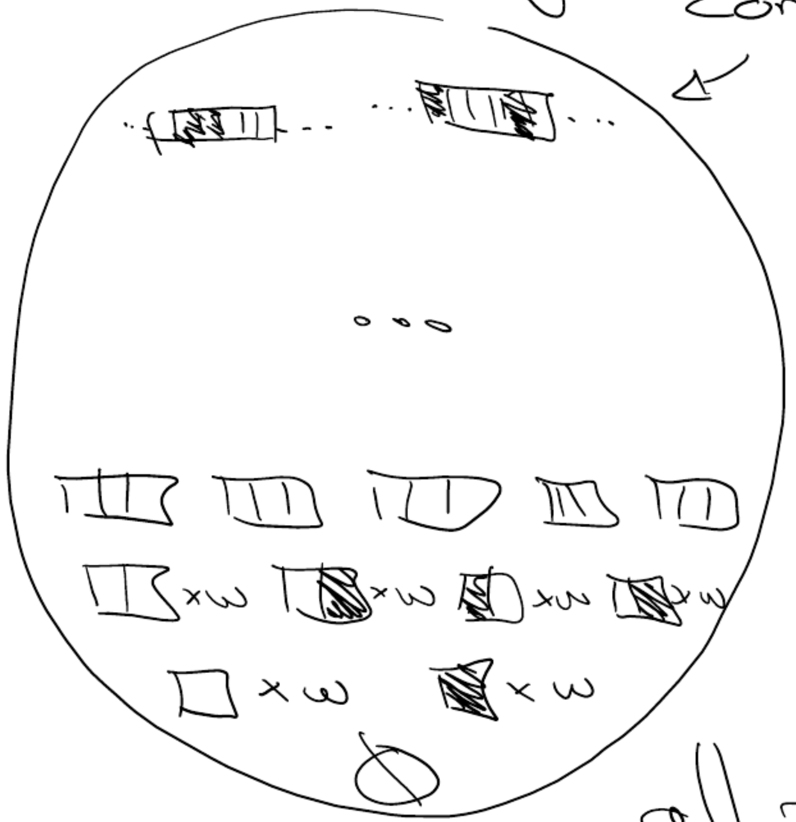
$\boxed{}_{10}\boxed{}_{11}\boxed{}_{12} \mapsto \boxed{}_{11}$

all partial funtions $\mathbb{Z}_S \to Q$, $f|_S \leq f$

Firsts Cellular Automata
Extension to Partial Configurations

Partial Configs

infinite configs



$F \leq G$

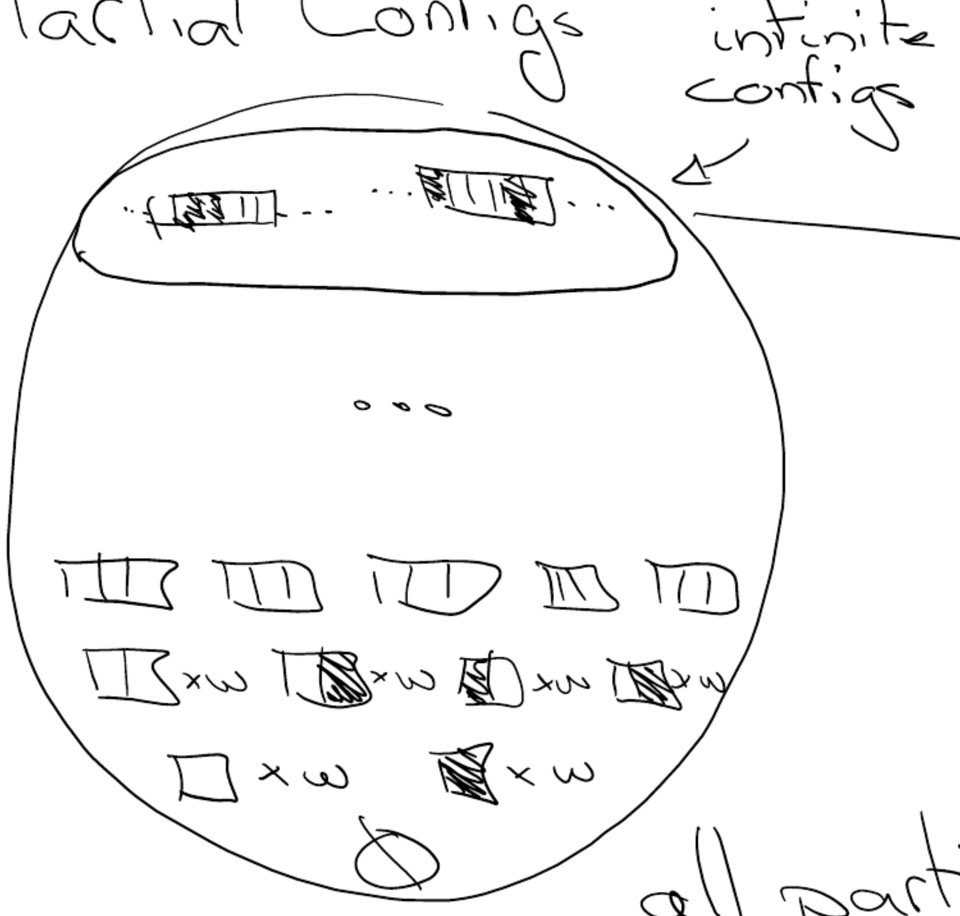
$\forall c,\ F(c) \leq G(c)$

Partial Configs

all partial funtions $\mathbb{Z}_r \to Q$, $f_{|S} \leq f$

Firsts Cellular Automata
Extension to Partial Configurations

Partial Configs



Δ

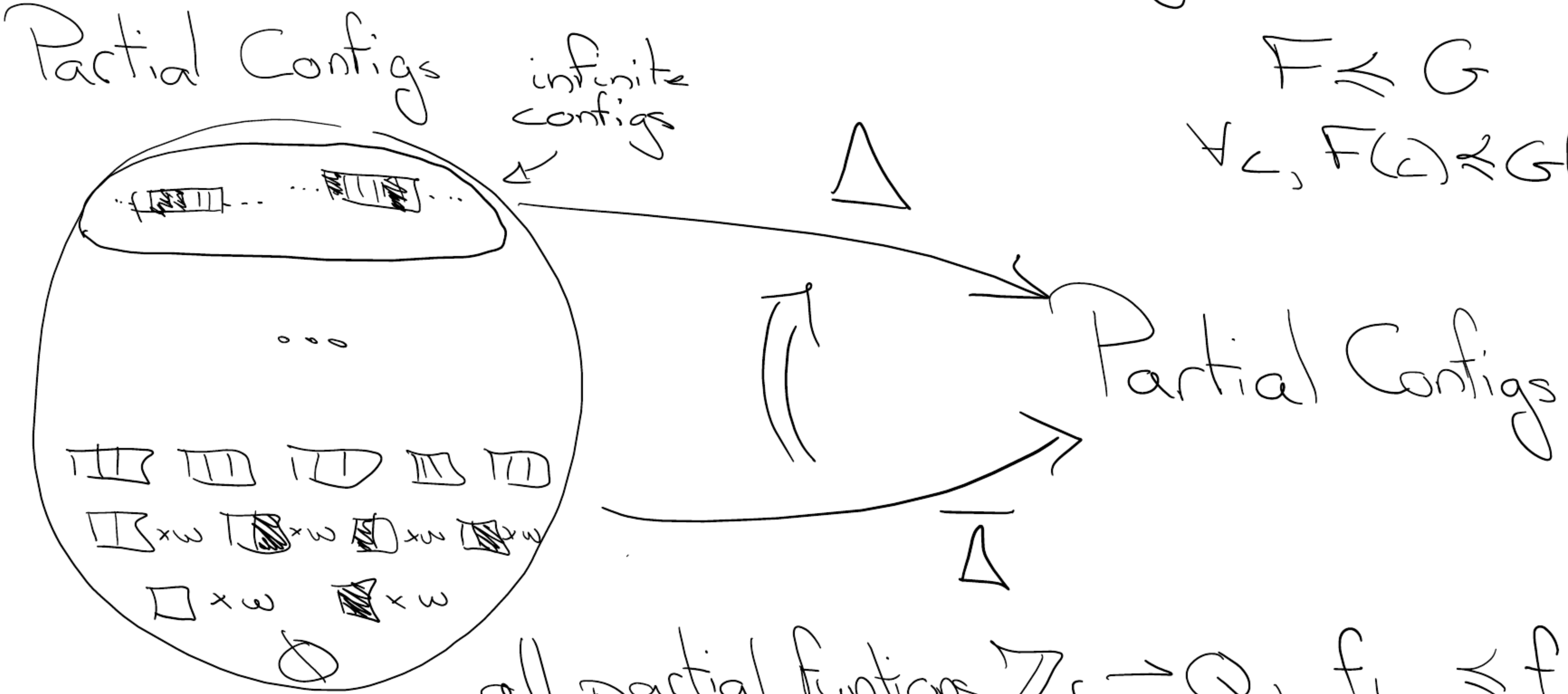
Partial Configs

$F \leq G$

$\forall c, F(c) \leq G(c)$

all partial funtions $\mathbb{Z}_r \to Q$, $f_{|S} \leq f$

Firsts Cellular Automata
Extension to Partial Configurations



$F \leq G$

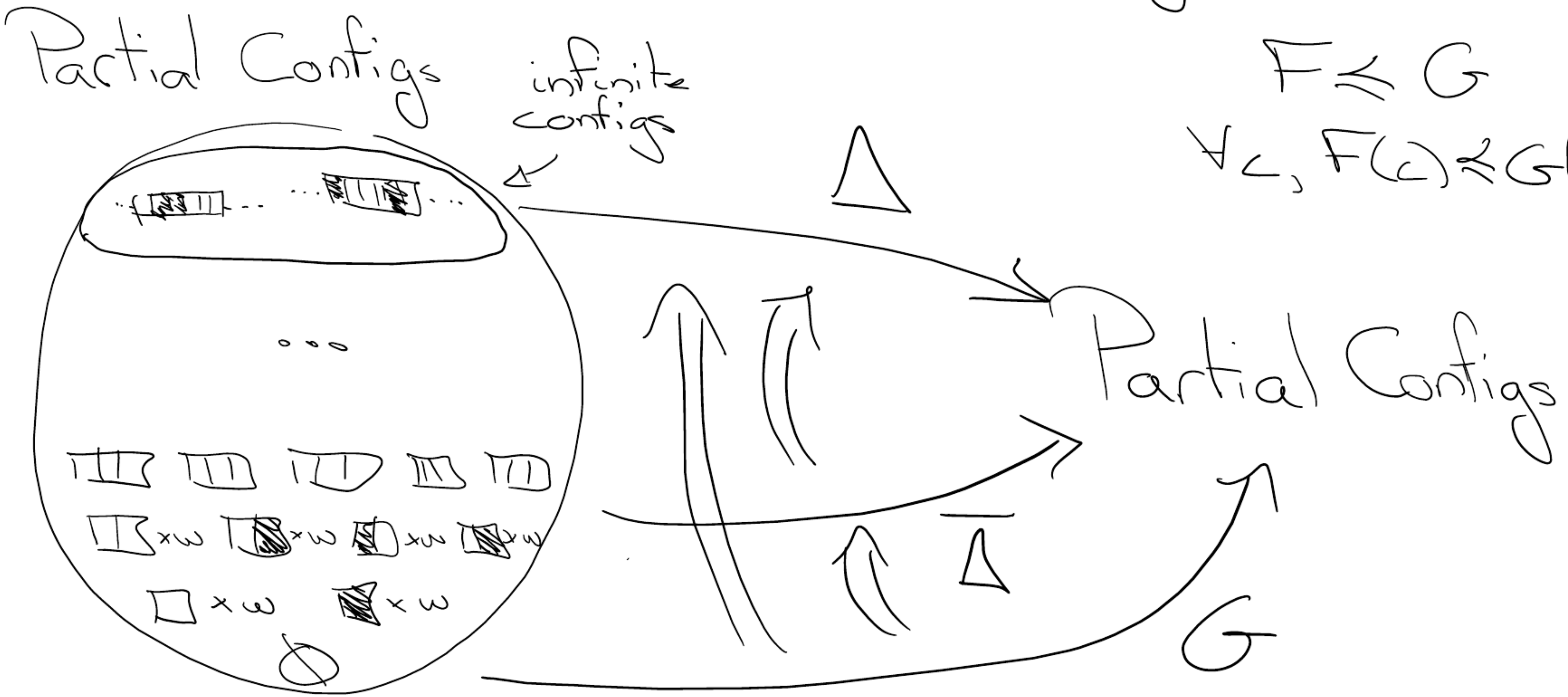
$\forall c,\ F(c) \leq G(c)$

all partial funtions $\mathbb{Z}_r \to Q$, $f|_S \leq f$

Firsts Cellular Automata Extension to Partial Configurations

$F \leq G$

$\forall c,\ F(c) \leq G(c)$



Firsts Cellular Automata Extension to Partial Configurations

$$\underline{\Delta} = \mathrm{Lan}_{I} \overleftrightarrow{\delta} \qquad \overline{\Delta} = \mathrm{Ran}_{J} \Delta$$

$$\begin{tikzcd}
\text{Partial} \arrow[dr, "\underline{\Delta}"] & \\
\text{Local} \arrow[u, "I"] \arrow[r, "\overleftrightarrow{\delta}"'] \arrow[ur, phantom, "\Uparrow"] & \text{Partial}
\end{tikzcd}
\qquad
\begin{tikzcd}
\text{Partial} \arrow[dr, "\overline{\Delta}"] & \\
\text{Global} \arrow[u, "J"] \arrow[r, "\Delta"'] \arrow[ur, phantom, "\Downarrow"] & \text{Partial}
\end{tikzcd}$$

More ~~Firsts~~ Cellular Automata

Extension to Partial Configurations

$$\underline{\Delta} = \mathrm{Lan}_I \overleftrightarrow{\delta} \qquad\qquad \overline{\Delta} = \mathrm{Ran}_J \Delta$$

$$\begin{tikzcd}
\text{Partial} \arrow[dr, "\underline{\Delta}"] & \\
\text{Local} \arrow[u, "I"] \arrow[r, "\overleftrightarrow{\delta}"'] \arrow[ur, phantom, "\Uparrow"] & \text{Partial}
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
\text{Partial} \arrow[dr, "\overline{\Delta}"] & \\
\text{Global} \arrow[u, "J"] \arrow[r, "\Delta"'] \arrow[ur, phantom, "\Downarrow"] & \text{Partial}
\end{tikzcd}$$

More ~~Firsts~~ Cellular Automata

Extension to Partial Configurations

$$\underline{\Delta} = \mathrm{Lan}_I \overleftrightarrow{\delta} \qquad\qquad \overline{\Delta} = \mathrm{Ran}_J \Delta$$

$$\begin{tikzcd}
\text{Partial} \arrow[dr, "\underline{\Delta}"] & \cong \\
\text{Local} \arrow[u, "I"] \arrow[r, "\overleftrightarrow{\delta}"'] \arrow[ur, phantom, "\Uparrow"] & \text{Partial}
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
\text{Partial} \arrow[dr, "\overline{\Delta}"] & \checkmark \\
\text{Global} \arrow[u, "J"] \arrow[r, "\Delta"'] \arrow[ur, phantom, "\Downarrow"] & \text{Partial}
\end{tikzcd}$$

More ~~Firsts~~ Cellular Automata

Extension to Partial Configurations

$$\underline{\Delta} = \mathrm{Lan}_I \overleftrightarrow{\delta}$$

$$\begin{tikzcd}
\text{Partial} \arrow[drr, bend left, "\underline{\Delta}"] & & \\
& \Uparrow & \\
\text{Local} \arrow[uu, "I"] \arrow[rr, "\overleftrightarrow{\delta}"'] & & \text{Partial}
\end{tikzcd}$$

I a simple inclusion

Θ a simple identity

Partial a simple poset

More ~~Firsts~~ Cellular Automata
Extension to Partial Configurations

$$\underline{\Delta} = \mathrm{Lan}_I \overleftrightarrow{\delta}$$

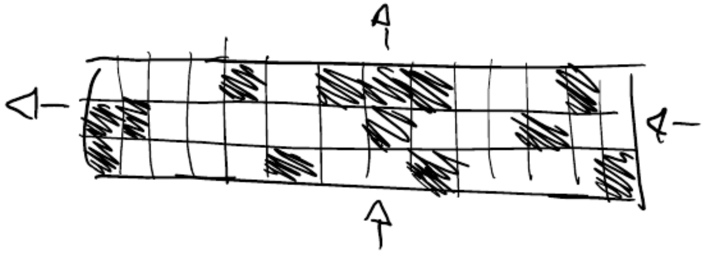
$$\begin{tikzcd}
\mathrm{Partial} \arrow[dr, "\underline{\Delta}"] & \\
\mathrm{Local} \arrow[u, "I"] \arrow[r, "\overleftrightarrow{\delta}"'] \arrow[ur, phantom, "\Uparrow"] & \mathrm{Partial}
\end{tikzcd}$$

I a simple inclusion

Θ a simple identity

Partial a simple poset

Torus



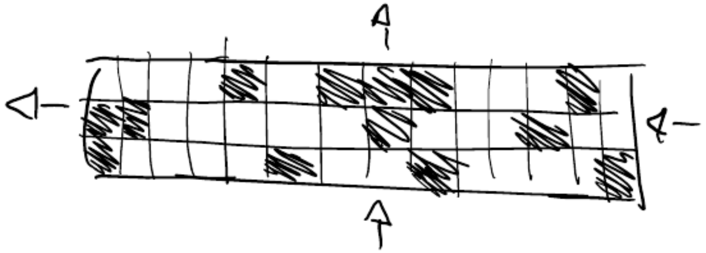
More ~~First~~ Cellular Automata
Extension to Partial Configurations

$$\underline{\Delta} = \mathrm{Lan}_I \overleftrightarrow{\delta}$$

I a simple inclusion
Θ a simple identity
Partial a simple post

$$\begin{tikzcd}
\text{Partial} \arrow[dr, bend left, "\underline{\Delta}"] & \\
\text{Local} \arrow[u, "I"] \arrow[r, "\overleftrightarrow{\delta}"'] \arrow[ur, phantom, "\Uparrow"] & \text{Partial}
\end{tikzcd}$$

Torus



More ~~Firsts~~ Cellular Automata
Extension to Partial Configurations

$$\underline{\Delta} = \mathrm{Lan}_{I} \overleftrightarrow{\delta}$$

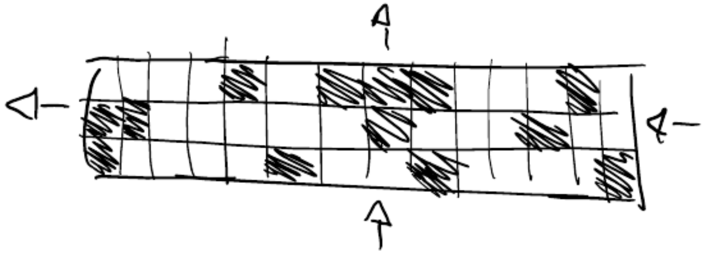
I a simple inclusion

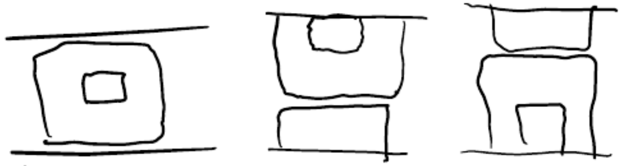
Θ a simple identity

Partial a simple post

$$\begin{tikzcd}
\mathrm{Partial} \arrow[dr, phantom, "\Uparrow"] \arrow[r, "\underline{\Delta}"] & \mathrm{Partial} \\
\mathrm{Local} \arrow[u, "I"] \arrow[ur, "\overleftrightarrow{\delta}"'] &
\end{tikzcd}$$

Torus





3 "inputs" $\longrightarrow$ 1 support

More ~~Firsts~~ Cellular Automata

Extension to Partial Configurations

$$\underline{\Delta} = \mathrm{Lan}_{\underline{I}} \overleftrightarrow{\delta}$$

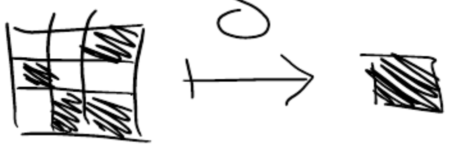
$\underline{I}$ a simple inclusion

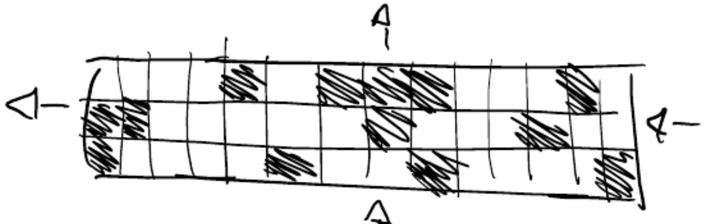
Θ a simple identity

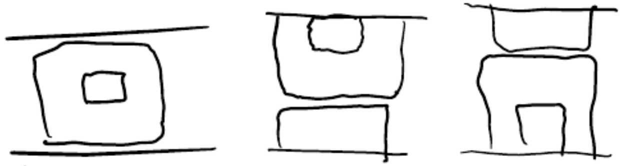
Partial a simple post

$$\begin{tikzcd} & \mathrm{Partial} \arrow[dr, bend left, "\underline{\Delta}"] \arrow[d, phantom, "\Uparrow"] & \\ \coprod_{p \in S} \mathrm{Local}(p) \arrow[rr, "\overleftrightarrow{\delta}"'] & & \mathrm{Partial} \end{tikzcd}$$

Torus







3 "inputs" ⟶ 1 support

More ~~First~~ Cellular Automata

Extension to Partial Configurations

$$\underline{\Delta} = \mathrm{Lan}_{\underline{I}} \overleftrightarrow{\delta}$$

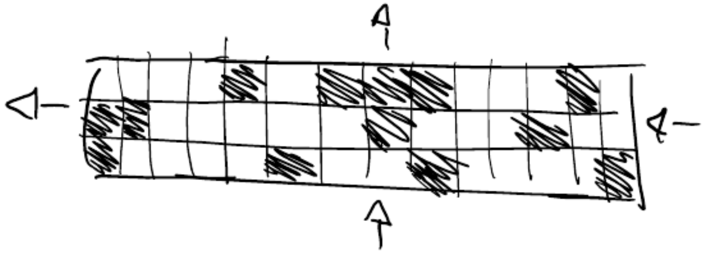
$\underline{I}$ a simple inclusion

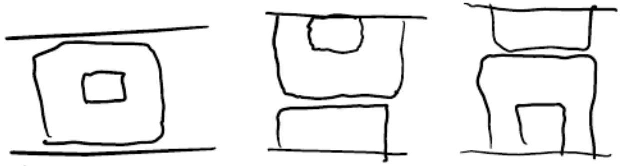
Θ a simple identity

Partial a simple post

$$\begin{tikzcd}
\text{Partial} \arrow[rr, bend left, "\underline{\Delta}"] & & \\
\coprod_{p \in S} \mathrm{Local}(p) \arrow[u, "\pi_2"] \arrow[rr, "\overleftrightarrow{\delta}"'] \arrow[ur, phantom, "\Uparrow"] & & \text{Partial}
\end{tikzcd}$$

Torus





3 "inputs" → 1 support

More ~~Firsts~~ Cellular Automata
Extension to Partial Configurations

$$\underline{\Delta} = \mathrm{Lan}_{\underline{I}} \overleftrightarrow{\delta}$$

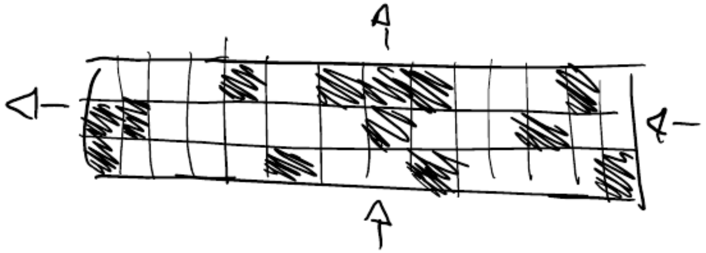
~~I a simple inclusion~~

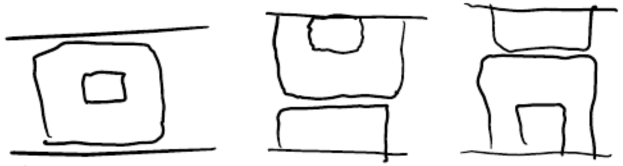
~~Θ a simple identity~~

Partial a simple post

$$\begin{tikzcd}
 & \text{Partial} \arrow[dr, "\underline{\Delta}"] & \\
\coprod_{p\in S} \mathrm{Local}(p) \arrow[ur, "\pi_2"] \arrow[rr, "\overleftrightarrow{\delta}"'] \arrow[rr, phantom, bend left=20, "\Uparrow"] & & \text{Partial}
\end{tikzcd}$$

Torus





3 "inputs" → 1 support

More ~~First~~ "Cellular Automata"

Extension to Partial Configurations

$$\underline{\Delta} = \mathrm{Lan}_{\underline{I}}\, \overleftrightarrow{\delta}$$

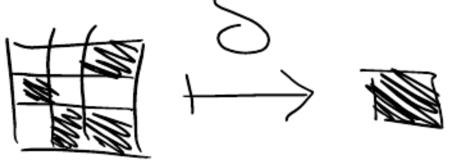
~~I a simple inclusion~~

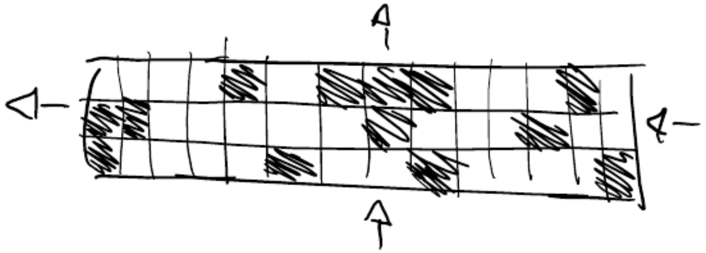
~~Θ a simple identity~~

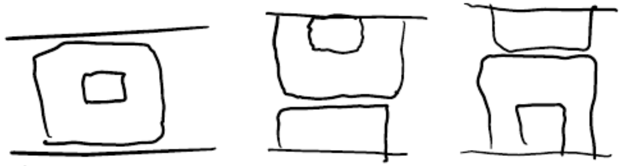
Partial a simple post

$$\begin{tikzcd}
\mathrm{Partial} \arrow[drr, "\underline{\Delta}"] & & \\
\coprod_{p\in S} \mathrm{Loc}(p) \arrow[u, "\pi_2"] \arrow[rr, "\overleftrightarrow{\delta}"'] \arrow[ur, phantom, "\Uparrow"] & & \mathrm{Partial}
\end{tikzcd}$$

Torus







3 "inputs" → 1 support

More ~~First~~ "Cellular Automata"

Extension to Partial Configurations

$$\underline{\Delta} = \mathrm{Lan}_I \overleftrightarrow{\delta}$$

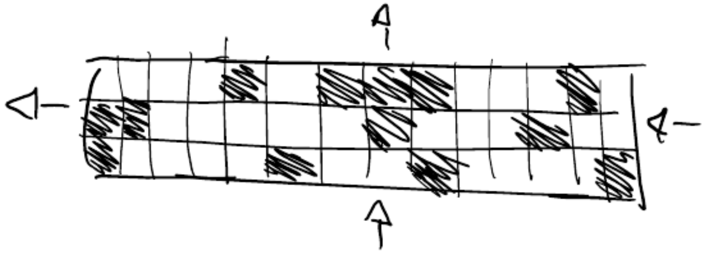
~~I a simple inclusion~~

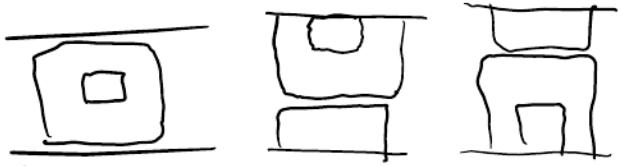
~~Θ a simple identity~~

~~Partial a simple post~~

$$\begin{tikzcd}
\mathrm{Partial} \arrow[drr, "\underline{\Delta}"] & & \\
\coprod_{p\in S} \mathrm{Local}(p) \arrow[u, "\pi_2"] \arrow[rr, "\overleftrightarrow{\delta}"'] \arrow[ur, phantom, "\Uparrow"] & & \mathrm{Partial}
\end{tikzcd}$$

Torus





3 "inputs" → 1 support

(a piece of)

CA & KE

Luidnel MAIGNAN
Antoine SPICHER
Alexandre FERNANDEZ

Global Transform°
LACL
UPEC