

LIFTING & LENSES

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OVERVIEW & MOTIVATION

01

algebraic weak factorisation systems

GENERALISE

orthogonal factorisation systems

OVERVIEW & MOTIVATION

01

algebraic weak factorisation systems

GENERALISE

orthogonal factorisation systems

Informally, an AWFS on \mathcal{C} consists of:

- compatible comonad L and monad R on \mathcal{C}^2

OVERVIEW & MOTIVATION

01

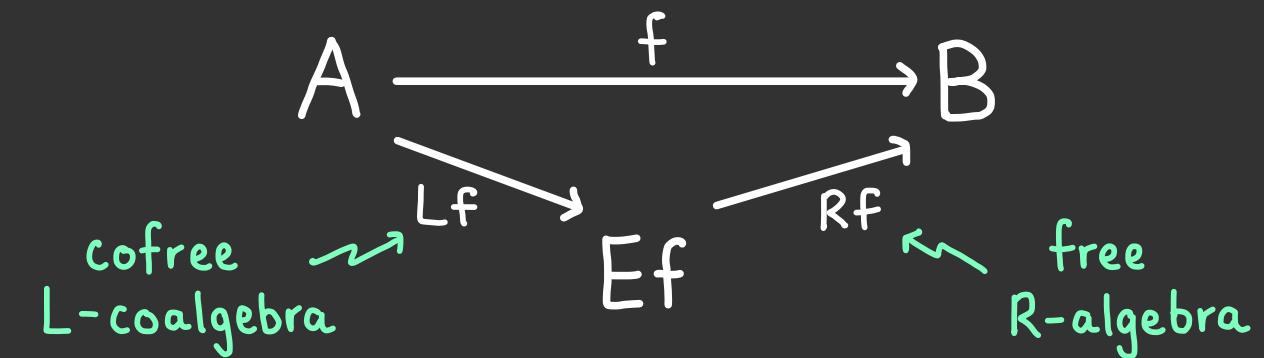
algebraic weak factorisation systems

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- functorial factorisation of each $f \in \mathcal{C}$



OVERVIEW & MOTIVATION

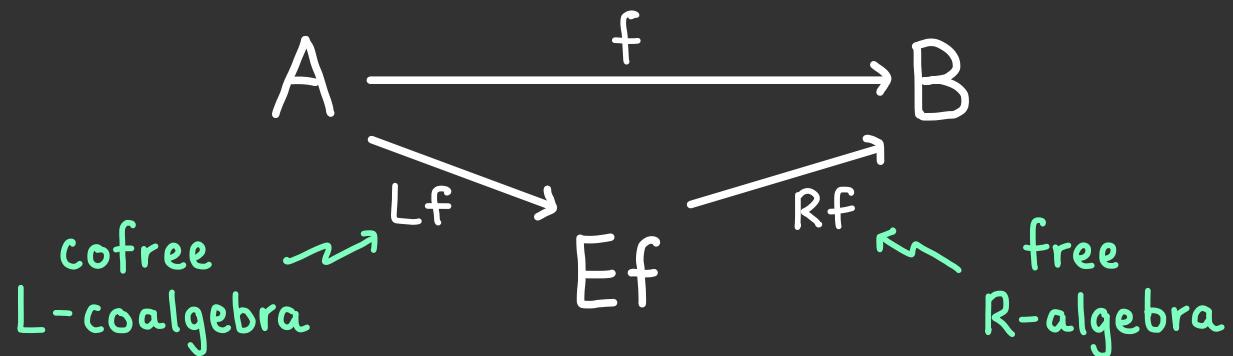
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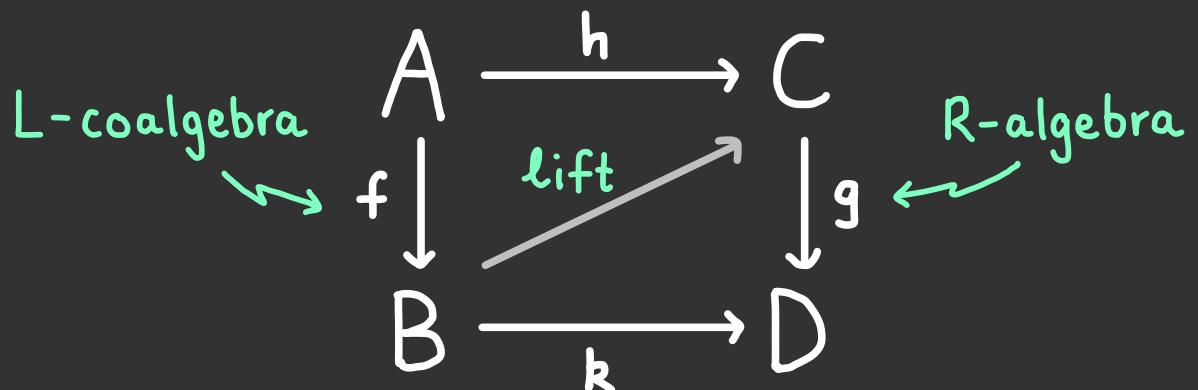
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- lifts of L -coalgebras against R -algebras



OVERVIEW & MOTIVATION

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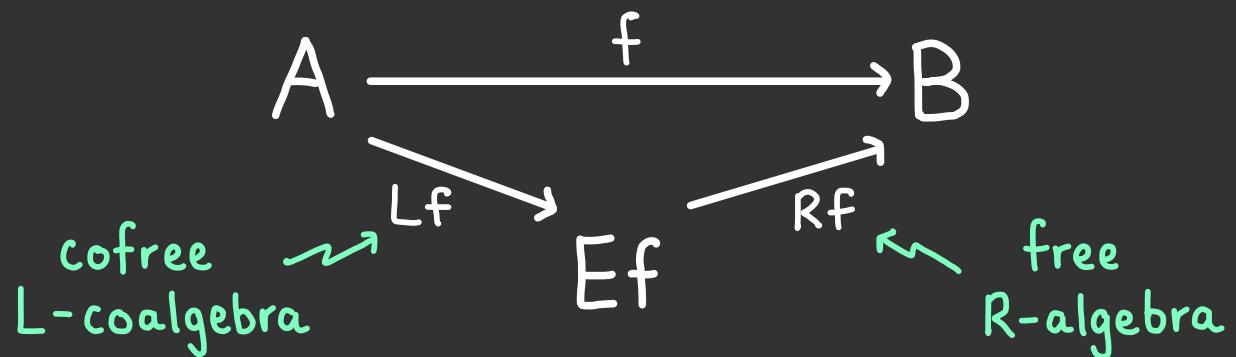
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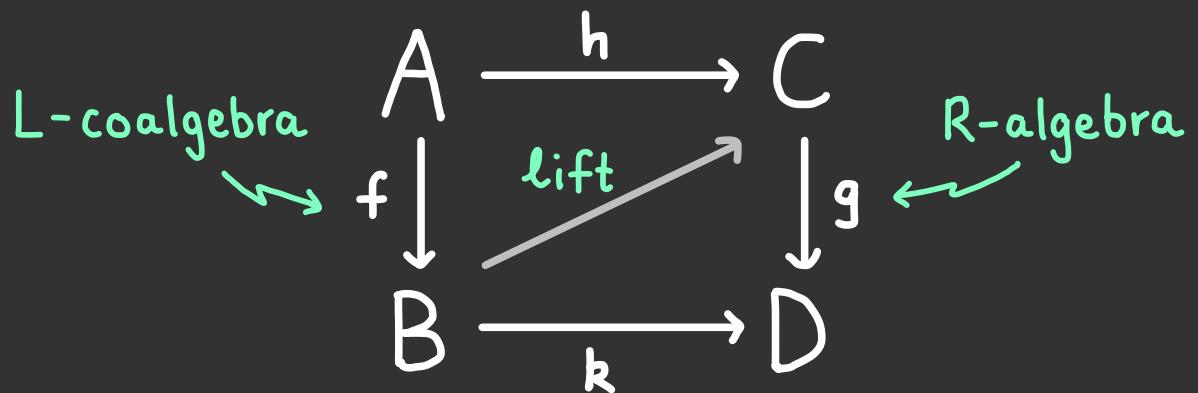
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- lifts of L -coalgebras against R -algebras



This talk: 2 related examples of AWFS

| L -coalgebra | R -algebra |
|----------------|-------------------|
| lari | split opfibration |
| ??? | delta lens |

SPLIT OPFIBRATIONS

o 2

A **split opfibration** is a functor equipped
with a lifting operation (splitting)

$$\begin{array}{ccc} A & \xrightarrow{\Phi(a,u)} & a' \\ f \downarrow & & \\ B & \xrightarrow{u} & b \end{array}$$

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4. Each lift $\varPhi(a,u)$ is opcartesian.

$$\begin{array}{ccc} a & \xrightarrow{\varPhi(a,u)} & a' \\ & \searrow \omega & \\ & & a'' \end{array}$$

$$\begin{array}{ccc} fa & \xrightarrow{u} & b \\ & \searrow f\omega & \swarrow v \\ & fa' & \end{array}$$

SPLIT OPFIBRATIONS

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$$\begin{array}{ccc} a & \xrightarrow{\varPhi(a,u)} & a' \\ \omega \searrow & \swarrow \exists! & \\ & a'' & \end{array}$$

$\downarrow f$

$$\begin{array}{ccc} fa & \xrightarrow{u} & b \\ f\omega \searrow & \swarrow v & \\ & fa' & \end{array}$$

FREE SPLIT OPFIBRATIONS

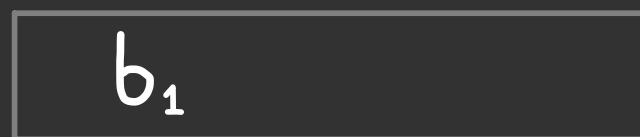
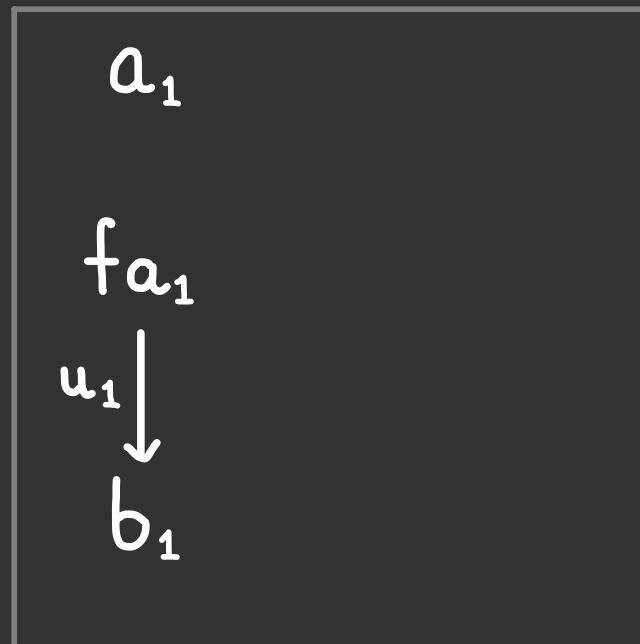
03

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$$\begin{array}{ccc} a_1 & \xrightarrow{\omega} & a_2 \\ f a_1 & \xrightarrow{f\omega} & f a_2 \\ u_1 \downarrow & \curvearrowright & \downarrow u_2 \\ b_1 & \xrightarrow{v} & b_2 \end{array}$$

comma
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The chosen opcartesian lifts are:

$$\begin{array}{ccc}
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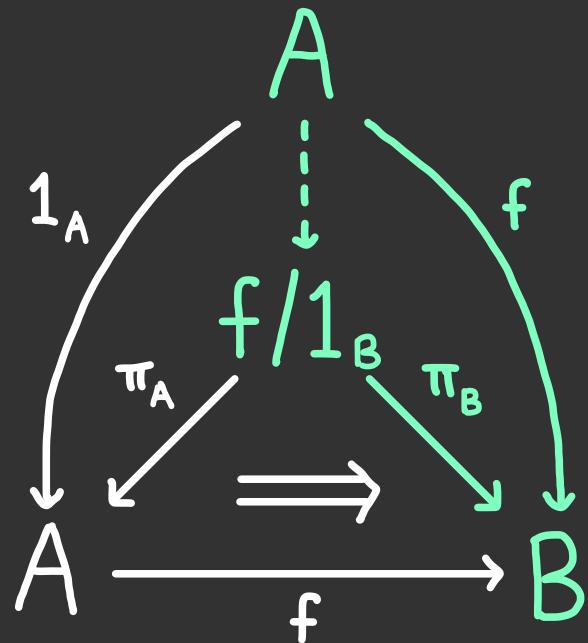
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THE AWFS FOR SPLIT OPFIBRATIONS

04

FACTORISATION

Every functor factorises into a (cofree) left-adjoint-right inverse followed by a (free) split opfibration.

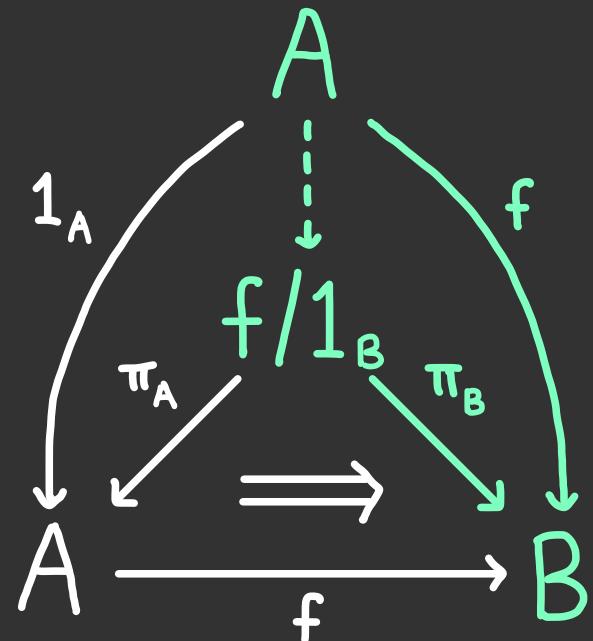


THE AWFS FOR SPLIT OPFIBRATIONS

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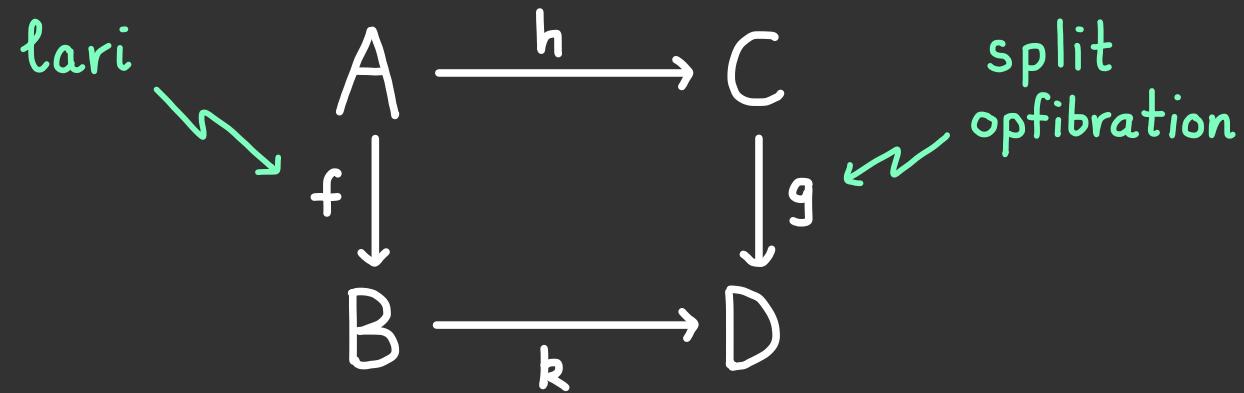
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Every functor factorises into a (cofree) left-adjoint-right inverse followed by a (free) split opfibration.



LIFTING

Given a commutative square in $\mathcal{C}\text{at}$



there is a canonical functor $j: B \rightarrow C$ such that $jf = h$ and $gj = k$.

E.g. Take $f: \{0 \rightarrow 2\} \rightarrow \{0 \rightarrow 1 \rightarrow 2\}$.

DELTA LENSES

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A delta lens is a functor equipped
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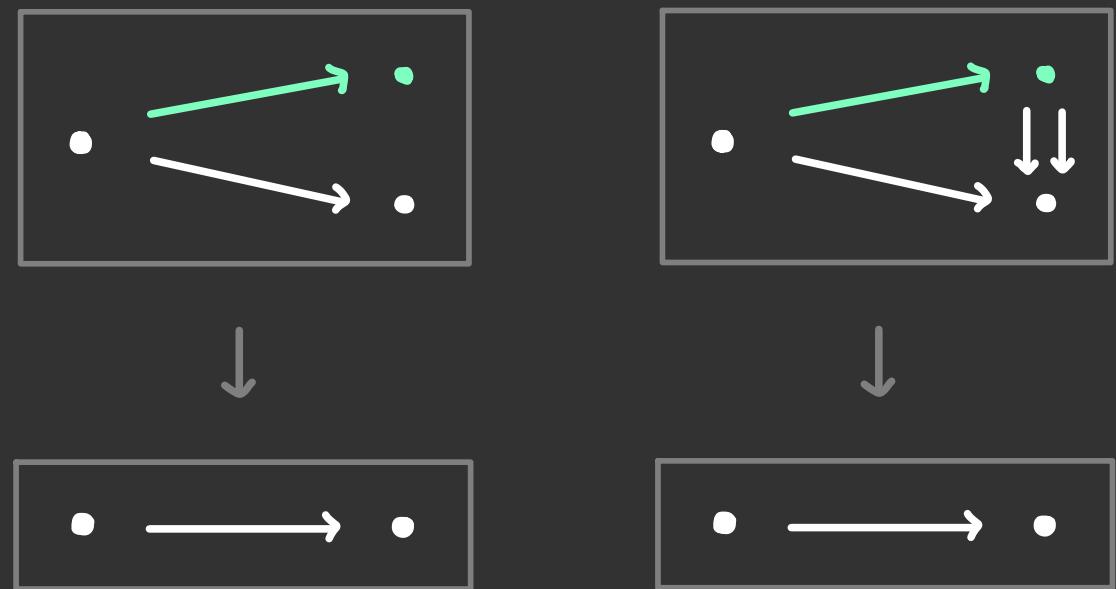
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Two simple examples of delta lenses which are not split opfibrations.



DELTA LENSES

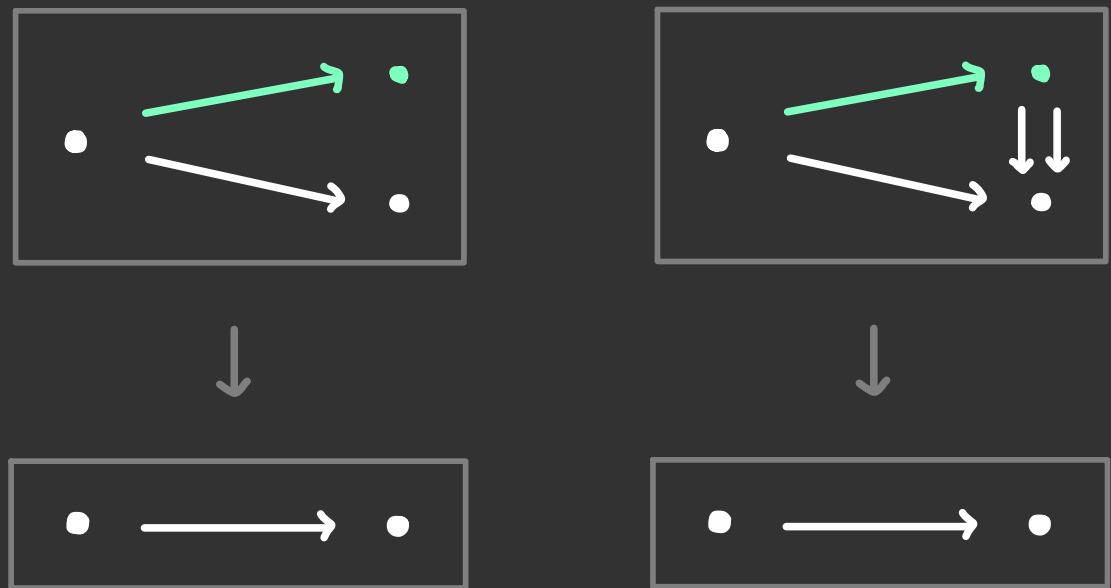
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Q: What is the free delta lens?

FREE DELTA LENSES (1)

06

The free delta lens $Rf: Ef \rightarrow B$ on a functor $f:A \rightarrow B$ has domain whose:

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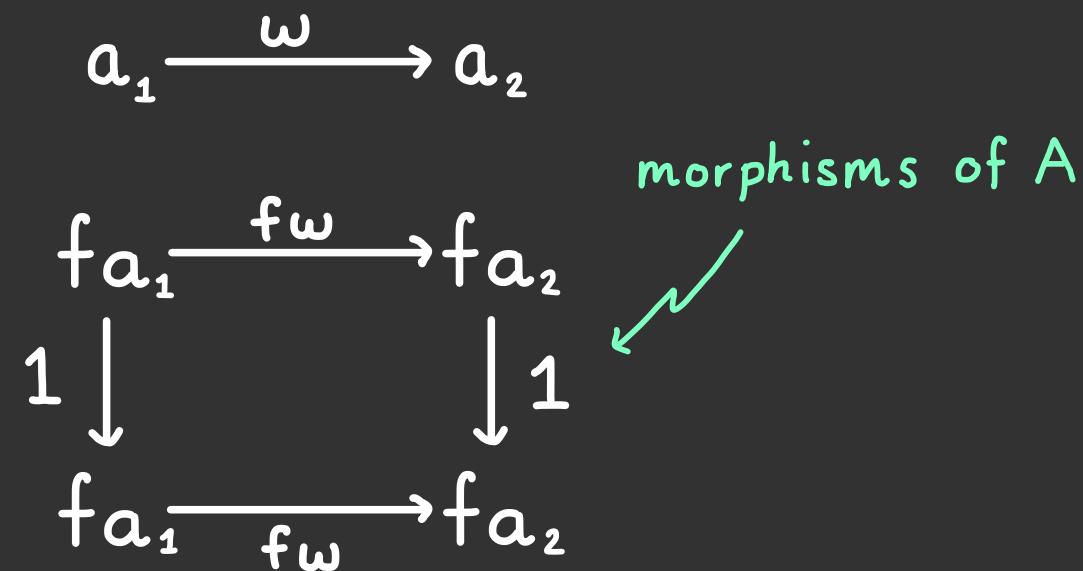
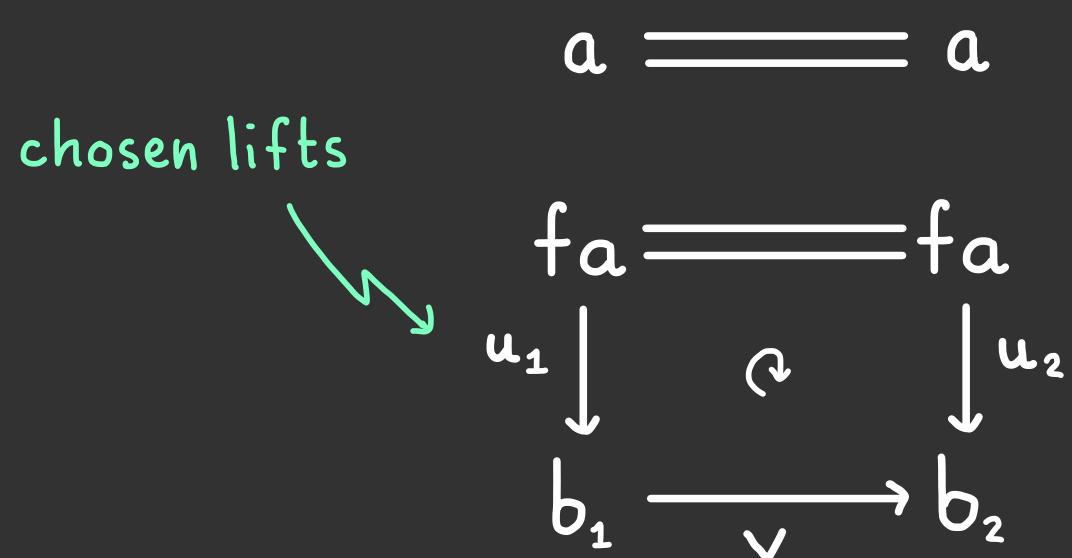
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- objects are pairs $(a \in A, u:fa \rightarrow b \in B)$
- morphisms are generated by the following:

$$\begin{array}{ccc} a & \xlongequal{\quad} & a \\ & \searrow \text{chosen lifts} & \\ fa & \xlongequal{\quad} & fa \\ u_1 \downarrow & \curvearrowright & \downarrow u_2 \\ b_1 & \xrightarrow{\quad} & b_2 \end{array}$$

$$\begin{array}{ccc} a_1 & \xrightarrow{\omega} & a_2 \\ & \searrow \text{morphisms of } A & \\ fa_1 & \xrightarrow{f\omega} & fa_2 \\ 1 \downarrow & & \downarrow 1 \\ fa_1 & \xrightarrow{fw} & fa_2 \end{array}$$

The functor Rf sends these generators to $v:b_1 \rightarrow b_2$ and $fw:fa_1 \rightarrow fa_2$, respectively.

FREE DELTA LENSES (2)

The free delta lens $Rf: Ef \rightarrow B$ on a functor $f:A \rightarrow B$ has domain whose:

- objects are pairs $(a \in A, u:fa \rightarrow b \in B)$
- morphisms $(a_1, u_1) \rightarrow (a_2, u_2)$ are given by the following two sorts:

$$a_1 = a_2$$

$$a_1 = a_1 \xrightarrow{\omega} a_2 = a_2$$

$$\begin{array}{ccc} fa_1 & = & fa_2 \\ u_1 \downarrow & \curvearrowright & \downarrow u_2 \\ b_1 & \xrightarrow{\vee} & b_2 \end{array}$$

$$\begin{array}{ccccc} fa_1 & = & fa_1 & \xrightarrow{f\omega} & fa_2 = fa_2 \\ u_1 \downarrow & \curvearrowright & 1 \downarrow & & \downarrow 1 \\ b_1 & \xrightarrow{\vee} & fa_1 & \xrightarrow{f\omega} & fa_2 \xrightarrow{u_2} b_2 \end{array}$$

The functor Rf sends these to $v:b_1 \rightarrow b_2$ and $u_2 \circ f\omega \circ v:fa_1 \rightarrow fa_2$, respectively.

THE AWFS FOR DELTA LENSES

07

FACTORISATION

$$A \xrightarrow{f} B$$

THE AWFS FOR DELTA LENSES

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FACTORISATION

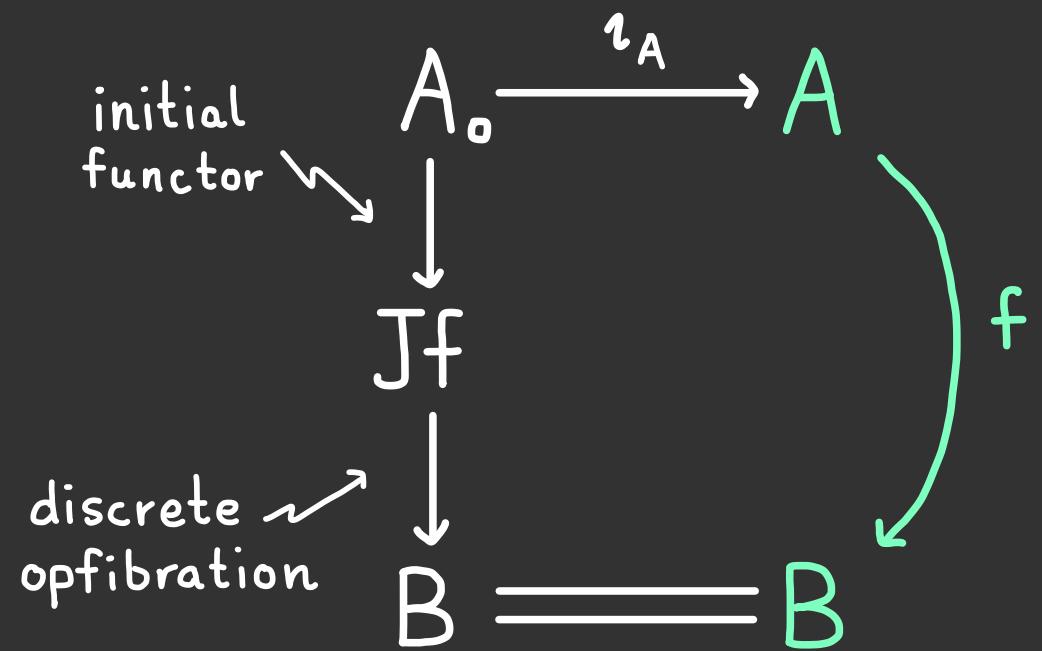
$$A_0 \xrightarrow{\iota_A} A$$

A diagram illustrating the factorisation of a space A_0 . An arrow labeled ι_A points from A_0 to A . From A , a curved arrow labeled f points down to B .

THE AWFS FOR DELTA LENSES

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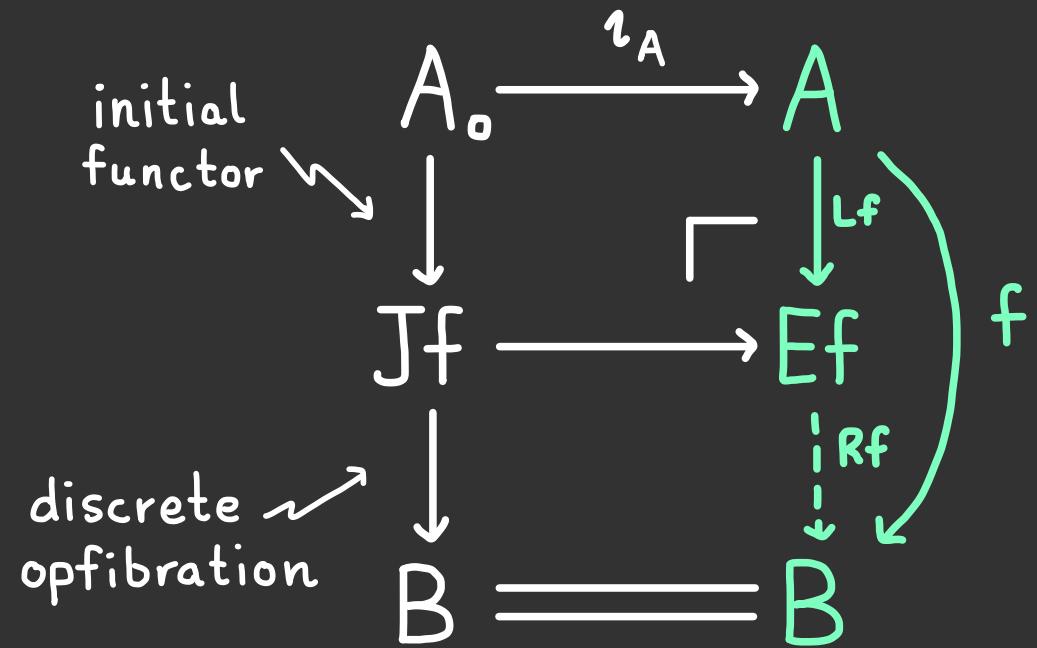
FACTORISATION



where $Jf = \sum_{a \in A_0} f_a / B$

THE AWFS FOR DELTA LENSES

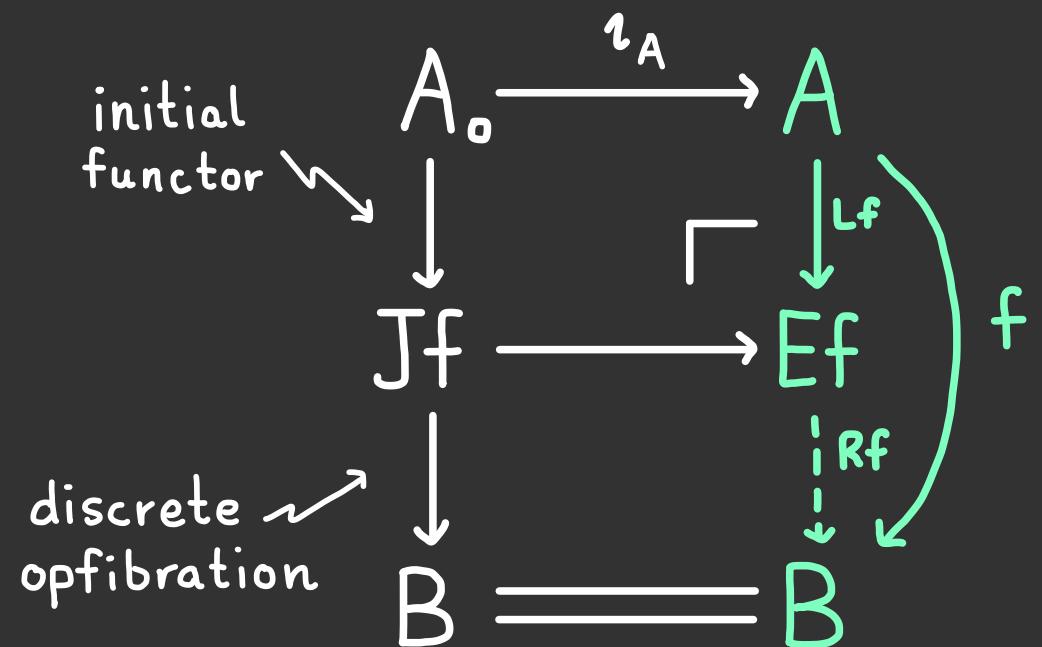
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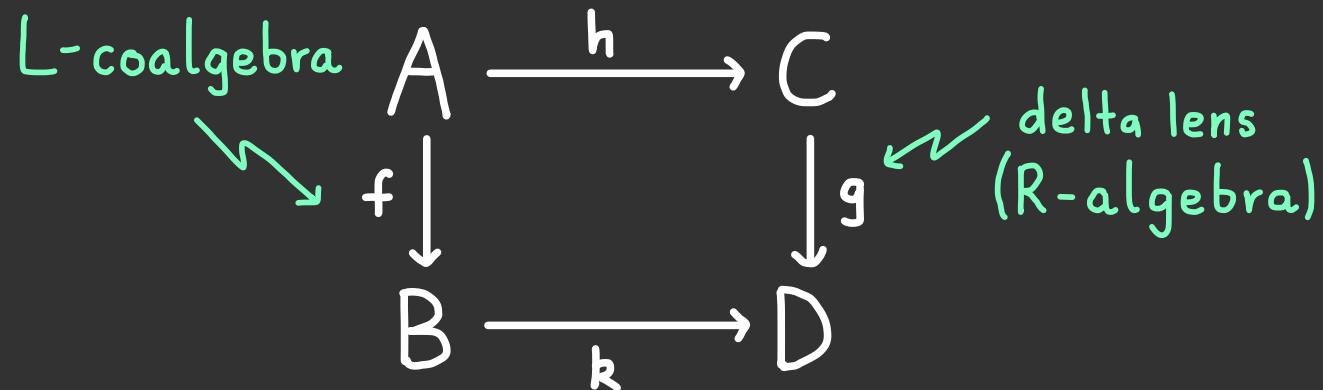
FACTORISATION



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LIFTING

Given a commutative square in \mathbf{Cat}

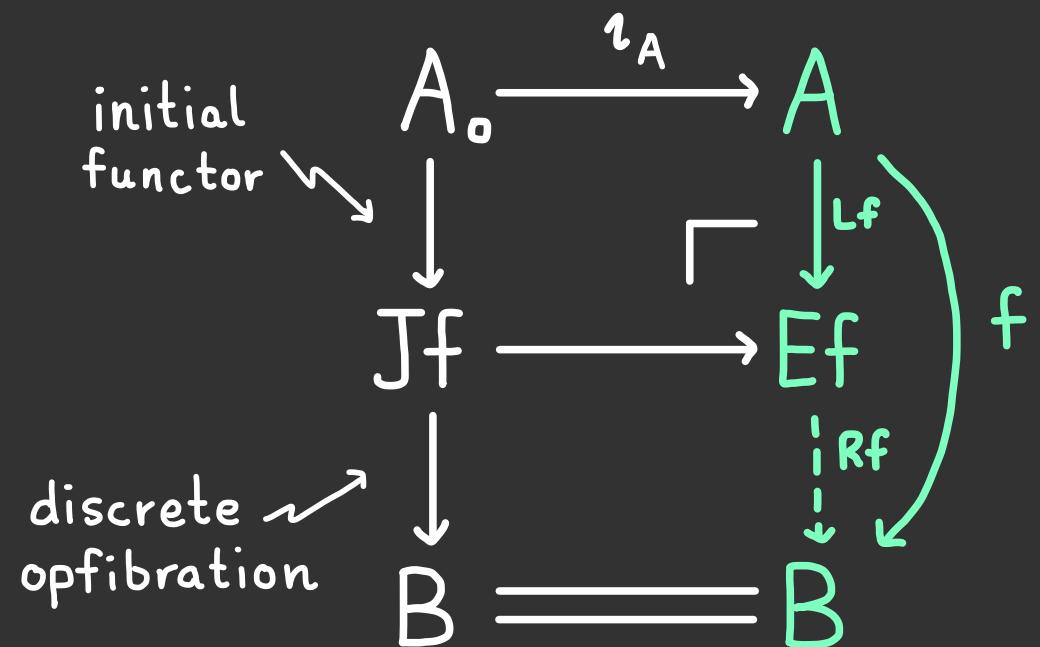


there is a canonical functor $j : B \rightarrow C$
such that $jf = h$ and $gj = k$.

THE AWFS FOR DELTA LENSES

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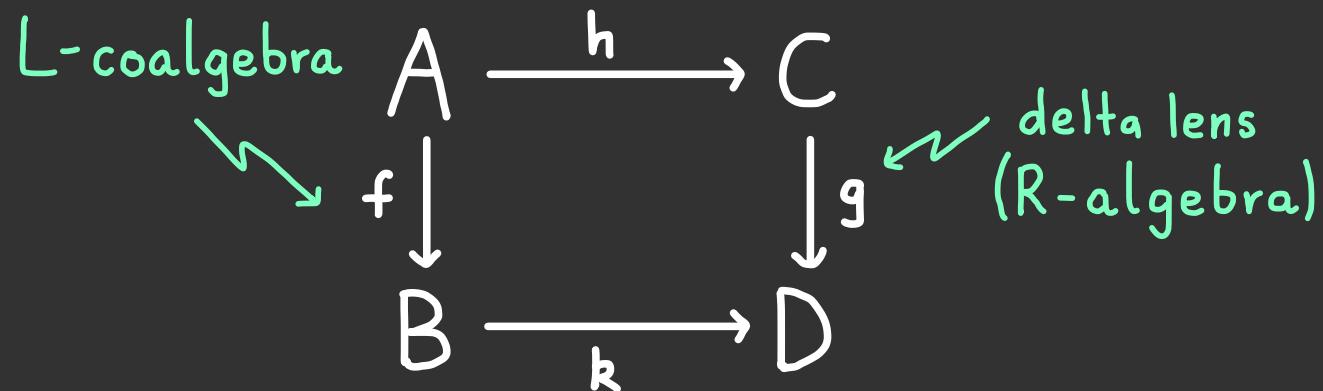
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LIFTING

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Q: What are the L -coalgebras?

COALGEBRAS ARE "STRUCTURED" LARIs

0 8

A L -coalgebra is an adjunction

$$\begin{array}{ccc} A & \begin{matrix} \xleftarrow{q} \\[-1ex] \xrightarrow{T} \\[-1ex] \xleftarrow{f} \end{matrix} & B \end{array} \quad q \circ f = \text{id}_A$$

such that if $q(u : b_1 \rightarrow b_2) \neq 1$, there is

a specified $\bar{q}u : b_1 \rightarrow fqb_1$ such that:

$$\begin{aligned} \bar{q}u \circ \varepsilon_{b_2} &= 1 \\ \varepsilon_{b_1} \circ f\bar{q}u \circ \bar{q}u &= u \end{aligned}$$

$$\begin{array}{ccc} fqb_1 & \xrightarrow{f\bar{q}u} & fqb_2 \\ \varepsilon_{b_1} \downarrow & \uparrow \bar{q}u & \downarrow \varepsilon_{b_2} \\ b_1 & \xrightarrow{u} & b_2 \end{array}$$

COALGEBRAS ARE "STRUCTURED" LARIs

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$$\begin{array}{ccccc} fqb_1 & \xrightarrow{\quad fqu \quad} & fqb_2 & & \\ \downarrow \varepsilon_{b_1} & \uparrow \bar{q}u & \downarrow \varepsilon_{b_2} & & \\ b_1 & \xrightarrow{\quad u \quad} & b_2 & & \end{array}$$

$$\boxed{\begin{aligned} \bar{q}u \circ \varepsilon_{b_2} &= 1 \\ \varepsilon_{b_1} \circ fqu \circ \bar{q}u &= u \end{aligned}}$$

The cofree L -coalgebra on $f : A \rightarrow B$ is

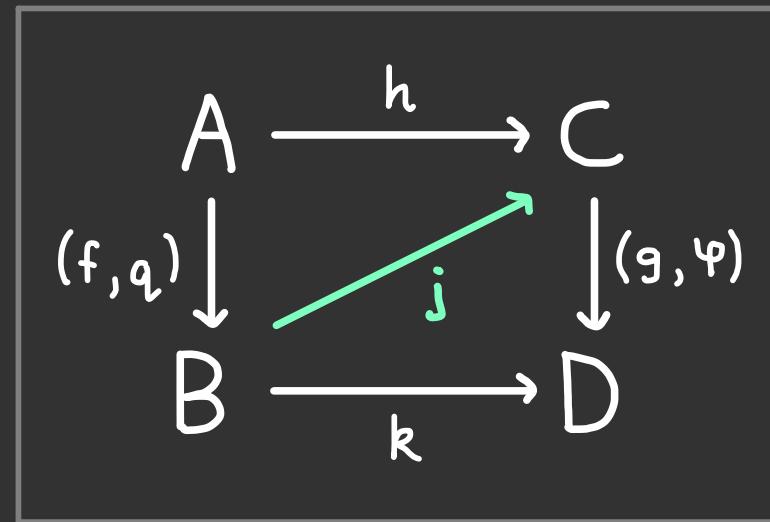
$$\begin{array}{ccc} a & \longleftarrow & (a, u) \\ A & \xleftarrow{\quad T \quad} & Ef \\ & \downarrow Lf & \\ a & \longmapsto & (a, 1_{fa}) \end{array}$$

with counit:

$$\begin{array}{ccc} a & \xlongequal{\quad} & a \\ fa & \xlongequal{\quad} & fa \\ 1_{fa} & \downarrow & \downarrow u \\ fa & \xrightarrow{\quad u \quad} & b \end{array}$$

HOW TO LIFT AGAINST DELTA LENSES (1)

09

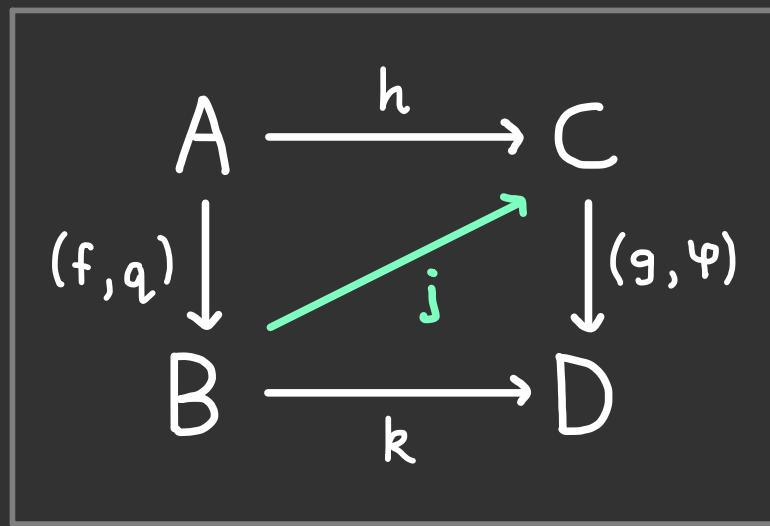


$$x \xrightarrow{u} y$$

HOW TO LIFT AGAINST DELTA LENSES (1)

09

$$qx \xrightarrow{q u} qy$$



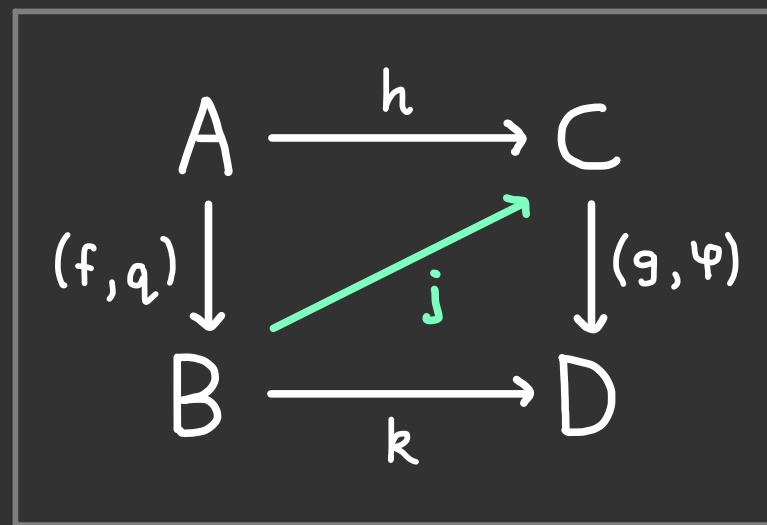
$$\begin{array}{ccc} f_q x & \xrightarrow{f_q u} & f_q y \\ \varepsilon_x \downarrow \uparrow \bar{q} u & & \downarrow \varepsilon_y \\ x & \xrightarrow{u} & y \end{array}$$

HOW TO LIFT AGAINST DELTA LENSES (1)

09

$$qx \xrightarrow{q^u} qy$$

$$hqx \xrightarrow{hq^u} hqy$$



$$fqx \xrightarrow{fq^u} fqy$$

$$\varepsilon_x \downarrow \uparrow \bar{q}u \quad \downarrow \varepsilon_y$$

$$x \xrightarrow{u} y$$

$$g(hqx) \quad g(hqy)$$

$$\parallel \quad \parallel$$

$$kfqx \quad kfqy$$

$$k\varepsilon_x \downarrow \uparrow k\bar{q}u \quad \downarrow k\varepsilon_y$$

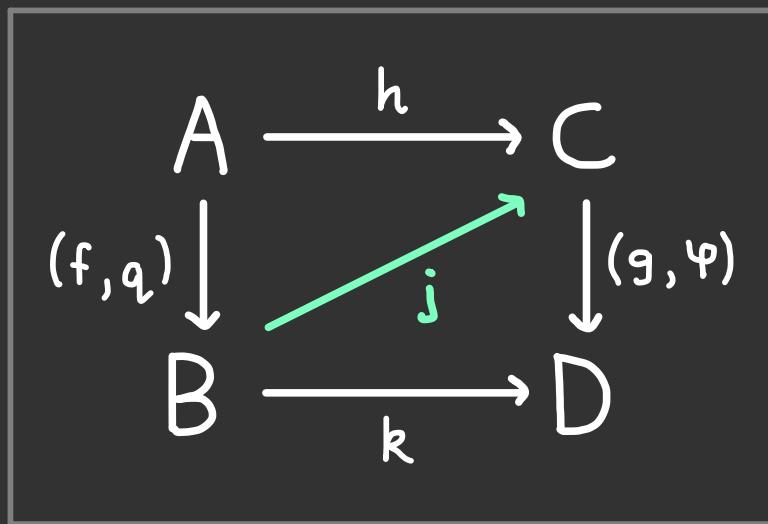
$$kx \quad ky$$

HOW TO LIFT AGAINST DELTA LENSES (1)

09

$$qx \xrightarrow{q u} qy$$

$$\begin{array}{ccc} hq x & \xrightarrow{h q u} & hq y \\ \downarrow \varphi(hq x, k \varepsilon_x) & \uparrow \varphi(j x, k \bar{q} u) & \downarrow \varphi(hq y, k \varepsilon_y) \\ j x & & j y \end{array}$$



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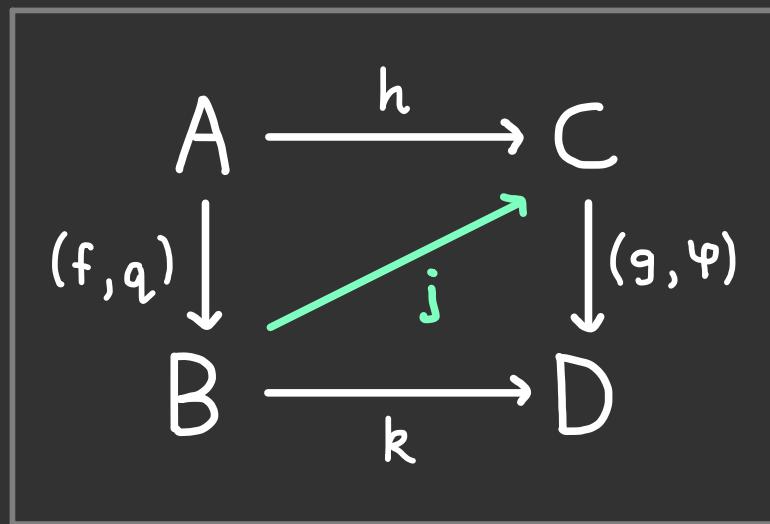
$$\begin{array}{ccc} g(hq x) & & g(hq y) \\ \parallel & & \parallel \\ kf q x & & kf q y \\ \downarrow k \varepsilon_x & \uparrow k \bar{q} u & \downarrow k \varepsilon_y \\ k x & & k y \end{array}$$

HOW TO LIFT AGAINST DELTA LENSES (1)

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$$qx \xrightarrow{q u} qy$$

$$\begin{array}{ccc} hq x & \xrightarrow{h q u} & hq y \\ \downarrow \varphi(hq x, k \varepsilon_x) & \uparrow \varphi(j x, k \bar{q} u) & \downarrow \varphi(hq y, k \varepsilon_y) \\ j x & \xrightarrow{j u} & j y \end{array}$$

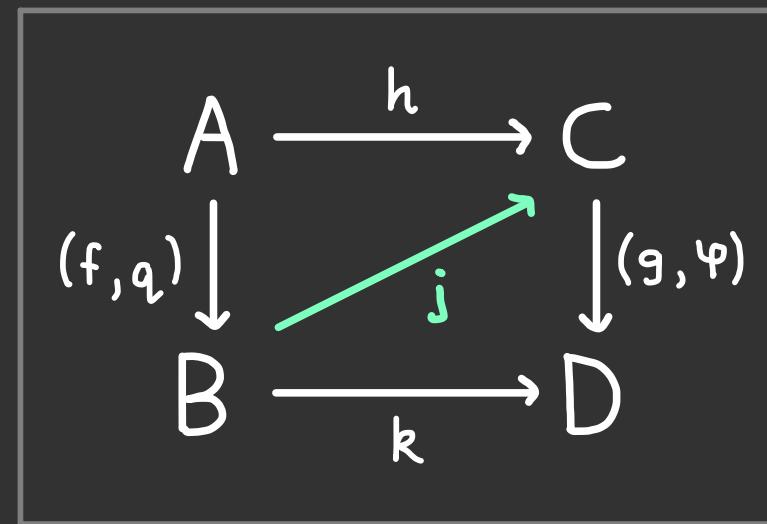


$$\begin{array}{ccc} f q x & \xrightarrow{f q u} & f q y \\ \varepsilon_x \downarrow \uparrow \bar{q} u & & \downarrow \varepsilon_y \\ x & \xrightarrow{u} & y \end{array}$$

$$\begin{array}{ccc} g(hq x) & & g(hq y) \\ \parallel & & \parallel \\ kf q x & & kf q y \\ \downarrow k \varepsilon_x & \uparrow k \bar{q} u & \downarrow k \varepsilon_y \\ kx & & ky \end{array}$$

HOW TO LIFT AGAINST DELTA LENSES (2)

09

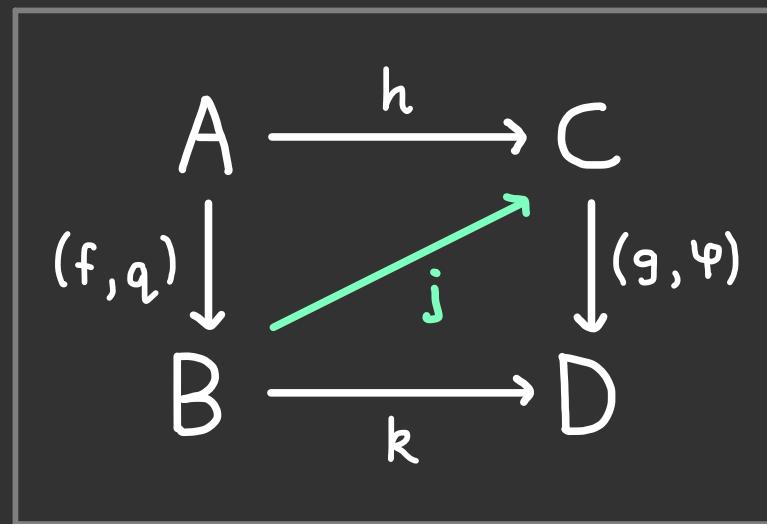


$$x \xrightarrow{u} y$$

HOW TO LIFT AGAINST DELTA LENSES (2)

09

$$q_x \xrightarrow{q^u = 1} q_y$$



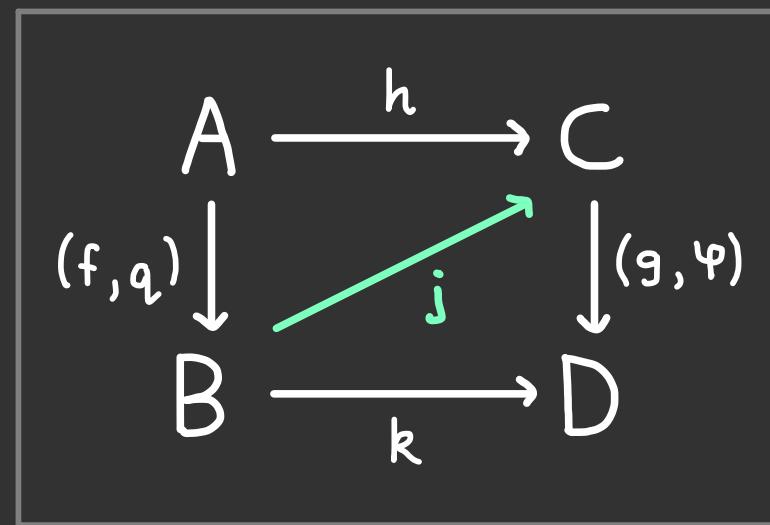
$$\begin{aligned} f q_x &= f q_y \\ \varepsilon_x \downarrow & \quad \downarrow \varepsilon_y \\ x &\xrightarrow{u} y \end{aligned}$$

HOW TO LIFT AGAINST DELTA LENSES (2)

09

$$q_x \xrightarrow{q^u=1} q_y$$

$$hq_x = hq_y$$



$$f_{q_x} = f_{q_y}$$

$$\varepsilon_x \downarrow \quad \downarrow \varepsilon_y$$

$$x \xrightarrow{u} y$$

$$g(hq_x) \quad g(hq_y)$$

$$\parallel \quad \parallel$$

$$kf_{q_x} = kf_{q_y}$$

$$k\varepsilon_x \downarrow \quad \downarrow k\varepsilon_y$$

$$kx \xrightarrow{ku} ky$$

HOW TO LIFT AGAINST DELTA LENSES (2)

09

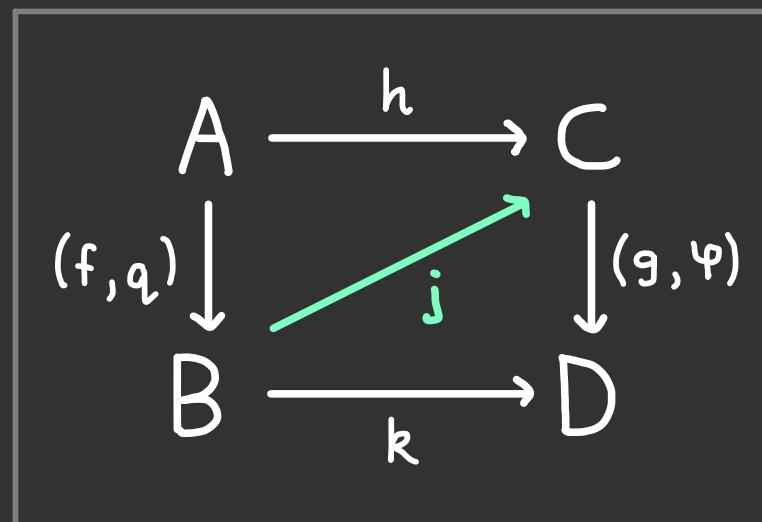
$$qx \xrightarrow{q^u=1} qy$$

$$hqx = hqy$$

$$\varphi(hq_x, k\varepsilon_x)$$

$$\downarrow \varphi(hq_y, k\varepsilon_y)$$

$$jx \xrightarrow{\varphi(jx, ku)} jy$$



$$fqx = fqy$$

$$\varepsilon_x \downarrow \quad \downarrow \varepsilon_y$$

$$x \xrightarrow{u} y$$

$$g(hq_x) \quad g(hq_y)$$

$$\parallel \quad \parallel$$

$$kfqx = kfqy$$

$$k\varepsilon_x \downarrow \quad \downarrow k\varepsilon_y$$

$$kx \xrightarrow{ku} ky$$

SUMMARY & FURTHER WORK

10

- We examined two examples of AWFS on Cat whose R -algebras were split opfibrations & delta lenses.
- We constructed explicitly the free delta lens on a functor.
- We characterised the coalgebras that delta lenses lift against as LARIs with extra structure.

SUMMARY & FURTHER WORK

- We examined two examples of AWFS on Cat whose R-algebras were split opfibrations & delta lenses.
- We constructed explicitly the free delta lens on a functor.
- We characterised the coalgebras that delta lenses lift against as LARIs with extra structure.

- The AWFS for delta lenses generalises to any "nice" category with OFS and idempotent comonad.
- Can assemble a double category of categories, functors, and delta lenses.

Check out the preprint:

arXiv:2305.02732