Combinatorics of super tableaux over signed alphabets

Nohra Hage

Université Catholique de Lille
Ecole du Numérique by FGES

June 5th, 2024

LHC days 2024, Nantes

I. Introduction and motivation

The Robinson–Schensted-Knuth correspondence

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1 & 3 & 3 & 5 \\ \hline 2 & 4 & 7 \\ \cline{1-3} 4 & 5 \\ \cline{1-2} 6 \\ \cline{1-1} \end{array} \leftsquigarrow_{S_r} 2 \;=\; \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 5 \\ \hline 2 & 3 & 7 \\ \cline{1-3} 4 & 4 \\ \cline{1-2} 5 \\ \cline{1-1} 6 \\ \cline{1-1} \end{array}$$

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1 & 3 & 3 & 5 \\ \hline 2 & 4 & 7 \\ \cline{1-3} 4 & 5 \\ \cline{1-2} 6 \\ \cline{1-1}\end{array} \leftsquigarrow_{S_r} 2 \;=\; \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 5 \\ \hline 2 & 3 & 7 \\ \cline{1-3} 4 & 4 \\ \cline{1-2} 5 \\ \cline{1-1} 6 \\ \cline{1-1}\end{array}$$

- **Robinson–Schensted correspondence** : $w = 231415$

$$w \longleftrightarrow (T(w), Q(w))$$

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1 & 3 & 3 & 5 \\ \hline 2 & 4 & 7 \\ \cline{1-3} 4 & 5 \\ \cline{1-2} 6 \\ \cline{1-1}\end{array} \leftsquigarrow_{S_r} 2 \;=\; \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 5 \\ \hline 2 & 3 & 7 \\ \cline{1-3} 4 & 4 \\ \cline{1-2} 5 \\ \cline{1-1} 6 \\ \cline{1-1}\end{array}$$

- **Robinson–Schensted correspondence** : $w = 231415$

$$w \longleftrightarrow (T(w), Q(w))$$

- $T(w)$ and $Q(w)$ are of **same-shape** tableaux :

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1&3&3&5\\\hline 2&4&7\\\cline{1-3} 4&5\\\cline{1-2} 6\\\cline{1-1}\end{array} \leftsquigarrow_{S_r} 2 = \begin{array}{|c|c|c|c|}\hline 1&2&3&5\\\hline 2&3&7\\\cline{1-3} 4&4\\\cline{1-2} 5\\\cline{1-1} 6\\\cline{1-1}\end{array}$$

- **Robinson–Schensted correspondence** : $w = 231415$

$$w \longleftrightarrow (T(w), Q(w))$$

- $T(w)$ and $Q(w)$ are of **same-shape** tableaux :

$(\emptyset\ ,\ \emptyset)$,

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1 & 3 & 3 & 5 \\ \hline 2 & 4 & 7 \\ \cline{1-3} 4 & 5 \\ \cline{1-2} 6 \\ \cline{1-1}\end{array} \leftsquigarrow_{S_r} 2 = \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 5 \\ \hline 2 & 3 & 7 \\ \cline{1-3} 4 & 4 \\ \cline{1-2} 5 \\ \cline{1-1} 6 \\ \cline{1-1}\end{array}$$

- **Robinson–Schensted correspondence** : $w = 231415$

$$w \longleftrightarrow (T(w), Q(w))$$

- $T(w)$ and $Q(w)$ are of **same-shape** tableaux :

$$(\emptyset\ ,\ \emptyset), \quad (\boxed{2}\ ,\ \boxed{1}),$$

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1&3&3&5\\\hline 2&4&7\\\cline{1-3} 4&5\\\cline{1-2} 6\\\cline{1-1}\end{array} \leftsquigarrow_{S_r} 2 = \begin{array}{|c|c|c|c|}\hline 1&2&3&5\\\hline 2&3&7\\\cline{1-3} 4&4\\\cline{1-2} 5\\\cline{1-1} 6\\\cline{1-1}\end{array}$$

- **Robinson–Schensted correspondence** : $w = 231415$

$$w \longleftrightarrow (T(w), Q(w))$$

- $T(w)$ and $Q(w)$ are of **same-shape** tableaux :

$$(\emptyset\ ,\ \emptyset),\quad (\boxed{2}\ ,\ \boxed{1}),\quad (\boxed{2\,|\,3}\ ,\ \boxed{1\,|\,2}),$$

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1&3&3&5\\\hline 2&4&7\\\cline{1-3} 4&5\\\cline{1-2} 6\\\cline{1-1}\end{array} \leftsquigarrow_{S_r} 2 = \begin{array}{|c|c|c|c|}\hline 1&2&3&5\\\hline 2&3&7\\\cline{1-3} 4&4\\\cline{1-2} 5\\\cline{1-1} 6\\\cline{1-1}\end{array}$$

- **Robinson–Schensted correspondence** : $w = 231415$

$$w \longleftrightarrow (T(w), Q(w))$$

- $T(w)$ and $Q(w)$ are of **same-shape** tableaux :

$$(\emptyset\ ,\ \emptyset), \quad (\boxed{2}\ ,\ \boxed{1}), \quad (\begin{array}{|c|c|}\hline 2&3\\\hline\end{array}\ ,\ \begin{array}{|c|c|}\hline 1&2\\\hline\end{array}), \quad (\begin{array}{|c|c|}\hline 1&3\\\hline 2\\\cline{1-1}\end{array}\ ,\ \begin{array}{|c|c|}\hline 1&2\\\hline 3\\\cline{1-1}\end{array}),$$

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1&3&3&5\\\hline 2&4&7\\\cline{1-3} 4&5\\\cline{1-2} 6\\\cline{1-1}\end{array} \leftsquigarrow_{S_r} 2 = \begin{array}{|c|c|c|c|}\hline 1&2&3&5\\\hline 2&3&7\\\cline{1-3} 4&4\\\cline{1-2} 5\\\cline{1-1} 6\\\cline{1-1}\end{array}$$

- **Robinson–Schensted correspondence** : $w = 231415$

$$w \longleftrightarrow (T(w), Q(w))$$

- $T(w)$ and $Q(w)$ are of **same-shape** tableaux :

$$(\emptyset\ ,\ \emptyset),\quad \left(\boxed{2}\ ,\ \boxed{1}\right),\quad \left(\begin{array}{|c|c|}\hline 2&3\\\hline\end{array}\ ,\ \begin{array}{|c|c|}\hline 1&2\\\hline\end{array}\right),\quad \left(\begin{array}{|c|c|}\hline 1&3\\\hline 2\\\cline{1-1}\end{array}\ ,\ \begin{array}{|c|c|}\hline 1&2\\\hline 3\\\cline{1-1}\end{array}\right),\quad \left(\begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2\\\cline{1-1}\end{array}\ ,\ \begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3\\\cline{1-1}\end{array}\right),$$

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1&3&3&5\\\hline 2&4&7\\\cline{1-3} 4&5\\\cline{1-2} 6\\\cline{1-1}\end{array} \leftsquigarrow_{S_r} 2 = \begin{array}{|c|c|c|c|}\hline 1&2&3&5\\\hline 2&3&7\\\cline{1-3} 4&4\\\cline{1-2} 5\\\cline{1-1} 6\\\cline{1-1}\end{array}$$

- **Robinson–Schensted correspondence** : $w = 231415$

$$w \longleftrightarrow (T(w), Q(w))$$

- $T(w)$ and $Q(w)$ are of **same-shape** tableaux :

$$(\emptyset , \emptyset), \quad \left(\begin{array}{|c|}\hline 2\\\hline\end{array} , \begin{array}{|c|}\hline 1\\\hline\end{array}\right), \quad \left(\begin{array}{|c|c|}\hline 2&3\\\hline\end{array} , \begin{array}{|c|c|}\hline 1&2\\\hline\end{array}\right), \quad \left(\begin{array}{|c|c|}\hline 1&3\\\hline 2\\\cline{1-1}\end{array} , \begin{array}{|c|c|}\hline 1&2\\\hline 3\\\cline{1-1}\end{array}\right), \quad \left(\begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2\\\cline{1-1}\end{array} , \begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3\\\cline{1-1}\end{array}\right),$$

$$\left(\begin{array}{|c|c|c|}\hline 1&1&4\\\hline 2&3\\\cline{1-2}\end{array} , \begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3&5\\\cline{1-2}\end{array}\right),$$

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1&3&3&5\\\hline 2&4&7\\\cline{1-3} 4&5\\\cline{1-2} 6\\\cline{1-1}\end{array} \leftsquigarrow_{S_r} 2 \;=\; \begin{array}{|c|c|c|c|}\hline 1&2&3&5\\\hline 2&3&7\\\cline{1-3} 4&4\\\cline{1-2} 5\\\cline{1-1} 6\\\cline{1-1}\end{array}$$

- **Robinson–Schensted correspondence** : $w = 231415$

$$w \longleftrightarrow (T(w), Q(w))$$

- $T(w)$ and $Q(w)$ are of **same-shape** tableaux :

$$(\emptyset\,,\,\emptyset),\quad \left(\boxed{2}\,,\,\boxed{1}\right),\quad \left(\begin{array}{|c|c|}\hline 2&3\\\hline\end{array}\,,\,\begin{array}{|c|c|}\hline 1&2\\\hline\end{array}\right),\quad \left(\begin{array}{|c|c|}\hline 1&3\\\hline 2\\\cline{1-1}\end{array}\,,\,\begin{array}{|c|c|}\hline 1&2\\\hline 3\\\cline{1-1}\end{array}\right),\quad \left(\begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2\\\cline{1-1}\end{array}\,,\,\begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3\\\cline{1-1}\end{array}\right),$$

$$\left(\begin{array}{|c|c|c|}\hline 1&1&4\\\hline 2&3\\\cline{1-2}\end{array}\,,\,\begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3&5\\\cline{1-2}\end{array}\right),\qquad \left(T(w) = \begin{array}{|c|c|c|c|}\hline 1&1&4&5\\\hline 2&3\\\cline{1-2}\end{array}\,,\; Q(w) = \begin{array}{|c|c|c|c|}\hline 1&2&4&6\\\hline 3&5\\\cline{1-2}\end{array}\right).$$

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|} \hline 1 & 3 & 3 & 5 \\ \hline 2 & 4 & 7 \\ \cline{1-3} 4 & 5 \\ \cline{1-2} 6 \\ \cline{1-1} \end{array} \;\leftsquigarrow_{S_r}\; 2 \;=\; \begin{array}{|c|c|c|c|} \hline 1 & 2 & 3 & 5 \\ \hline 2 & 3 & 7 \\ \cline{1-3} 4 & 4 \\ \cline{1-2} 5 \\ \cline{1-1} 6 \\ \cline{1-1} \end{array}$$

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1 & 3 & 3 & 5 \\ \hline 2 & 4 & 7 \\ \cline{1-3} 4 & 5 \\ \cline{1-2} 6 \\ \cline{1-1} \end{array} \leftsquigarrow_{S_r} 2 \;=\; \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 5 \\ \hline 2 & 3 & 7 \\ \cline{1-3} 4 & 4 \\ \cline{1-2} 5 \\ \cline{1-1} 6 \\ \cline{1-1} \end{array}$$

- **Two-rowed array :**

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 & 1 & 5 \end{pmatrix}$$

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1 & 3 & 3 & 5 \\ \hline 2 & 4 & 7 \\ \cline{1-3} 4 & 5 \\ \cline{1-2} 6 \\ \cline{1-1} \end{array} \leftsquigarrow_{S_r} 2 \; = \; \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 5 \\ \hline 2 & 3 & 7 \\ \cline{1-3} 4 & 4 \\ \cline{1-2} 5 \\ \cline{1-1} 6 \\ \cline{1-1} \end{array}$$

- **Two-rowed array :**

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 & 1 & 5 \end{pmatrix}$$

- **Robinson–Schensted–Knuth (RSK) correspondence** :

$$W \longleftrightarrow (T(W), Q(W))$$

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1 & 3 & 3 & 5 \\ \hline 2 & 4 & 7 & \\ \cline{1-3} 4 & 5 & & \\ \cline{1-2} 6 & & & \\ \cline{1-1}\end{array} \leftsquigarrow_{S_r} 2 = \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 5 \\ \hline 2 & 3 & 7 & \\ \cline{1-3} 4 & 4 & & \\ \cline{1-2} 5 & & & \\ \cline{1-1} 6 & & & \\ \cline{1-1}\end{array}$$

- **Two-rowed array :**

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 & 1 & 5 \end{pmatrix}$$

- **Robinson–Schensted–Knuth (RSK) correspondence** :

$$W \longleftrightarrow (T(W), Q(W))$$

- $T(W)$ and $Q(W)$ are of **same-shape** tableaux :

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1 & 3 & 3 & 5 \\ \hline 2 & 4 & 7 \\ \cline{1-3} 4 & 5 \\ \cline{1-2} 6 \\ \cline{1-1} \end{array} \leftsquigarrow_{S_r} 2 = \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 5 \\ \hline 2 & 3 & 7 \\ \cline{1-3} 4 & 4 \\ \cline{1-2} 5 \\ \cline{1-1} 6 \\ \cline{1-1} \end{array}$$

- **Two-rowed array :**

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 & 1 & 5 \end{pmatrix}$$

- **Robinson–Schensted–Knuth (RSK) correspondence** :

$$W \longleftrightarrow (T(W), Q(W))$$

- $T(W)$ and $Q(W)$ are of **same-shape** tableaux :

$$(\emptyset \,,\, \emptyset),$$

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1&3&3&5\\\hline 2&4&7\\\cline{1-3} 4&5\\\cline{1-2} 6\\\cline{1-1}\end{array} \leftsquigarrow_{S_r} 2 = \begin{array}{|c|c|c|c|}\hline 1&2&3&5\\\hline 2&3&7\\\cline{1-3} 4&4\\\cline{1-2} 5\\\cline{1-1} 6\\\cline{1-1}\end{array}$$

- **Two-rowed array :**

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 & 1 & 5 \end{pmatrix}$$

- **Robinson–Schensted–Knuth (RSK) correspondence** :

$$W \longleftrightarrow (T(W), Q(W))$$

- $T(W)$ and $Q(W)$ are of **same-shape** tableaux :

$$(\emptyset\ ,\ \emptyset), \quad (\boxed{2}\ ,\ \boxed{1}),$$

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1&3&3&5\\\hline 2&4&7\\\cline{1-3} 4&5\\\cline{1-2} 6\\\cline{1-1}\end{array} \leftsquigarrow_{S_r} 2 \;=\; \begin{array}{|c|c|c|c|}\hline 1&2&3&5\\\hline 2&3&7\\\cline{1-3} 4&4\\\cline{1-2} 5\\\cline{1-1} 6\\\cline{1-1}\end{array}$$

- **Two-rowed array :**

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 & 1 & 5 \end{pmatrix}$$

- **Robinson–Schensted–Knuth (RSK) correspondence** :

$$W \longleftrightarrow (T(W), Q(W))$$

- $T(W)$ and $Q(W)$ are of **same-shape** tableaux :

$$(\emptyset\,,\ \emptyset),\quad \left(\boxed{2}\,,\ \boxed{1}\right),\quad \left(\begin{array}{|c|c|}\hline 2&3\\\hline\end{array}\,,\ \begin{array}{|c|c|}\hline 1&1\\\hline\end{array}\right),$$

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1 & 3 & 3 & 5 \\ \hline 2 & 4 & 7 \\ \cline{1-3} 4 & 5 \\ \cline{1-2} 6 \\ \cline{1-1}\end{array} \leftsquigarrow_{S_r} 2 \;=\; \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 5 \\ \hline 2 & 3 & 7 \\ \cline{1-3} 4 & 4 \\ \cline{1-2} 5 \\ \cline{1-1} 6 \\ \cline{1-1}\end{array}$$

- **Two-rowed array :**

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 & 1 & 5 \end{pmatrix}$$

- **Robinson–Schensted–Knuth (RSK) correspondence** :

$$W \longleftrightarrow (T(W), Q(W))$$

- $T(W)$ and $Q(W)$ are of **same-shape** tableaux :

$$(\emptyset\,,\,\emptyset),\quad \left(\boxed{2}\,,\,\boxed{1}\right),\quad \left(\begin{array}{|c|c|}\hline 2 & 3 \\ \hline\end{array}\,,\,\begin{array}{|c|c|}\hline 1 & 1 \\ \hline\end{array}\right),\quad \left(\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1}\end{array}\,,\,\begin{array}{|c|c|}\hline 1 & 1 \\ \hline 2 \\ \cline{1-1}\end{array}\right),$$

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1&3&3&5\\\hline 2&4&7\\\cline{1-3} 4&5\\\cline{1-2} 6\\\cline{1-1}\end{array} \rightsquigarrow_{S_r} 2 = \begin{array}{|c|c|c|c|}\hline 1&2&3&5\\\hline 2&3&7\\\cline{1-3} 4&4\\\cline{1-2} 5\\\cline{1-1} 6\\\cline{1-1}\end{array}$$

- **Two-rowed array :**

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 & 1 & 5 \end{pmatrix}$$

- **Robinson–Schensted–Knuth (RSK) correspondence** :

$$W \longleftrightarrow (T(W), Q(W))$$

- $T(W)$ and $Q(W)$ are of **same-shape** tableaux :

$$(\emptyset\,,\,\emptyset),\quad \left(\boxed{2}\,,\,\boxed{1}\right),\quad \left(\begin{array}{|c|c|}\hline 2&3\\\hline\end{array}\,,\,\begin{array}{|c|c|}\hline 1&1\\\hline\end{array}\right),\quad \left(\begin{array}{|c|c|}\hline 1&3\\\hline 2\\\cline{1-1}\end{array}\,,\,\begin{array}{|c|c|}\hline 1&1\\\hline 2\\\cline{1-1}\end{array}\right),\quad \left(\begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2\\\cline{1-1}\end{array}\,,\,\begin{array}{|c|c|c|}\hline 1&1&2\\\hline 2\\\cline{1-1}\end{array}\right),$$

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1&3&3&5\\\hline 2&4&7\\\cline{1-3} 4&5\\\cline{1-2} 6\\\cline{1-1}\end{array} \;\leftsquigarrow_{S_r}\; 2 \;=\; \begin{array}{|c|c|c|c|}\hline 1&2&3&5\\\hline 2&3&7\\\cline{1-3} 4&4\\\cline{1-2} 5\\\cline{1-1} 6\\\cline{1-1}\end{array}$$

- **Two-rowed array :**

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 & 1 & 5 \end{pmatrix}$$

- **Robinson–Schensted–Knuth (RSK) correspondence** :

$$W \longleftrightarrow (T(W), Q(W))$$

- $T(W)$ and $Q(W)$ are of **same-shape** tableaux :

$$(\emptyset\,,\,\emptyset),\quad \left(\boxed{2}\,,\,\boxed{1}\right),\quad \left(\begin{array}{|c|c|}\hline 2&3\\\hline\end{array}\,,\,\begin{array}{|c|c|}\hline 1&1\\\hline\end{array}\right),\quad \left(\begin{array}{|c|c|}\hline 1&3\\\hline 2\\\cline{1-1}\end{array}\,,\,\begin{array}{|c|c|}\hline 1&1\\\hline 2\\\cline{1-1}\end{array}\right),\quad \left(\begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2\\\cline{1-1}\end{array}\,,\,\begin{array}{|c|c|c|}\hline 1&1&2\\\hline 2\\\cline{1-1}\end{array}\right),$$

$$\left(\begin{array}{|c|c|c|}\hline 1&1&4\\\hline 2&3\\\cline{1-2}\end{array}\,,\,\begin{array}{|c|c|c|}\hline 1&1&2\\\hline 2&3\\\cline{1-2}\end{array}\right),$$

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1&3&3&5\\\hline 2&4&7\\\cline{1-3} 4&5\\\cline{1-2} 6\\\cline{1-1}\end{array} \leftsquigarrow_{S_r} 2 = \begin{array}{|c|c|c|c|}\hline 1&2&3&5\\\hline 2&3&7\\\cline{1-3} 4&4\\\cline{1-2} 5\\\cline{1-1} 6\\\cline{1-1}\end{array}$$

- **Two-rowed array :**

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 & 1 & 5 \end{pmatrix}$$

- **Robinson–Schensted–Knuth (RSK) correspondence** :

$$W \longleftrightarrow (T(W), Q(W))$$

- $T(W)$ and $Q(W)$ are of **same-shape** tableaux :

$$(\emptyset\,,\,\emptyset),\quad \left(\begin{array}{|c|}\hline 2\\\hline\end{array}\,,\,\begin{array}{|c|}\hline 1\\\hline\end{array}\right),\quad \left(\begin{array}{|c|c|}\hline 2&3\\\hline\end{array}\,,\,\begin{array}{|c|c|}\hline 1&1\\\hline\end{array}\right),\quad \left(\begin{array}{|c|c|}\hline 1&3\\\hline 2\\\cline{1-1}\end{array}\,,\,\begin{array}{|c|c|}\hline 1&1\\\hline 2\\\cline{1-1}\end{array}\right),\quad \left(\begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2\\\cline{1-1}\end{array}\,,\,\begin{array}{|c|c|c|}\hline 1&1&2\\\hline 2\\\cline{1-1}\end{array}\right),$$

$$\left(\begin{array}{|c|c|c|}\hline 1&1&4\\\hline 2&3\\\cline{1-2}\end{array}\,,\,\begin{array}{|c|c|c|}\hline 1&1&2\\\hline 2&3\\\cline{1-2}\end{array}\right),\qquad \left(T(W) = \begin{array}{|c|c|c|c|}\hline 1&1&4&5\\\hline 2&3\\\cline{1-2}\end{array}\,,\ Q(W) = \begin{array}{|c|c|c|c|}\hline 1&1&2&4\\\hline 2&3\\\cline{1-2}\end{array}\right).$$

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1&3&3&5\\\hline 2&4&7\\\cline{1-3} 4&5\\\cline{1-2} 6\\\cline{1-1}\end{array} \leftsquigarrow_{S_r} 2 = \begin{array}{|c|c|c|c|}\hline 1&2&3&5\\\hline 2&3&7\\\cline{1-3} 4&4\\\cline{1-2} 5\\\cline{1-1} 6\\\cline{1-1}\end{array}$$

- **Two-rowed array** :

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 & 1 & 5 \end{pmatrix}$$

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1 & 3 & 3 & 5 \\ \hline 2 & 4 & 7 & \\ \cline{1-3} 4 & 5 & & \\ \cline{1-2} 6 & & & \\ \cline{1-1}\end{array} \leftsquigarrow_{S_r} 2 = \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 5 \\ \hline 2 & 3 & 7 & \\ \cline{1-3} 4 & 4 & & \\ \cline{1-2} 5 & & & \\ \cline{1-1} 6 & & & \\ \cline{1-1}\end{array}$$

- **Two-rowed array** :

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 & 1 & 5 \end{pmatrix}$$

- **Inverse of a two-rowed array** :

$$W^{\text{inv}} = \begin{pmatrix} 1 & 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 1 & 2 & 4 \end{pmatrix}$$

The Robinson–Schensted-Knuth correspondence

- **Schensted's insertion**, (Schensted, '61) :

$$\begin{array}{|c|c|c|c|}\hline 1 & 3 & 3 & 5 \\ \hline 2 & 4 & 7 & \\ \cline{1-3} 4 & 5 & & \\ \cline{1-2} 6 & & & \\ \cline{1-1}\end{array} \leftsquigarrow_{S_r} 2 \; = \; \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 5 \\ \hline 2 & 3 & 7 & \\ \cline{1-3} 4 & 4 & & \\ \cline{1-2} 5 & & & \\ \cline{1-1} 6 & & & \\ \cline{1-1}\end{array}$$

- **Two-rowed array** :

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 & 1 & 5 \end{pmatrix}$$

- **Inverse of a two-rowed array** :

$$W^{\text{inv}} = \begin{pmatrix} 1 & 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 1 & 2 & 4 \end{pmatrix}$$

- **Symmetry property**, (Viennot, '77, Fulton, '97) :

$$W^{\text{inv}} \xleftrightarrow{RSK} (Q(W), T(W))$$

Signed alphabets

Signed alphabets

- **Signed alphabet** $(\mathcal{S}, ||.||)$:
  - $\mathcal{S}$ a finite or countable totally ordered set,
  - $||.|| : \mathcal{S} \longrightarrow \mathbb{Z}_2 = \{0, 1\}$ be any map,
  - $\mathcal{S}_0 = \{a \in \mathcal{S} \mid ||a|| = 0\}$ and $\mathcal{S}_1 = \{a \in \mathcal{S} \mid ||a|| = 1\}$, $\rightsquigarrow \mathcal{S} = \mathcal{S}_0 \cup \mathcal{S}_1$.

Signed alphabets

- **Signed alphabet** $(\mathcal{S}, ||.||)$:
  - $\mathcal{S}$ a finite or countable totally ordered set,
  - $||.|| : \mathcal{S} \longrightarrow \mathbb{Z}_2 = \{0, 1\}$ be any map,
  - $\mathcal{S}_0 = \{a \in \mathcal{S} \mid ||a|| = 0\}$ and $\mathcal{S}_1 = \{a \in \mathcal{S} \mid ||a|| = 1\}, \rightsquigarrow \mathcal{S} = \mathcal{S}_0 \cup \mathcal{S}_1$.
- A question is to obtain a super version of the RSK correspondence (with symmetry) on super tableaux :

Signed alphabets

- **Signed alphabet** $(\mathcal{S}, ||.||)$:
  - $\mathcal{S}$ a finite or countable totally ordered set,
  - $||.|| : \mathcal{S} \longrightarrow \mathbb{Z}_2 = \{0, 1\}$ be any map,
  - $\mathcal{S}_0 = \{a \in \mathcal{S} \mid ||a|| = 0\}$ and $\mathcal{S}_1 = \{a \in \mathcal{S} \mid ||a|| = 1\}$, $\rightsquigarrow \mathcal{S} = \mathcal{S}_0 \cup \mathcal{S}_1$.
- A question is to obtain a super version of the RSK correspondence (with symmetry) on super tableaux :
  - (Bonetti, Senato, Venezia, '88) : a super version using super Schensted's insertion
    - (La Scala, Nardozza, Senato, '06) : It is not symmetric

Signed alphabets

- **Signed alphabet** $(\mathcal{S}, ||.||)$:
  - $\mathcal{S}$ a finite or countable totally ordered set,
  - $||.|| : \mathcal{S} \longrightarrow \mathbb{Z}_2 = \{0, 1\}$ be any map,
  - $\mathcal{S}_0 = \{a \in \mathcal{S} \mid ||a|| = 0\}$ and $\mathcal{S}_1 = \{a \in \mathcal{S} \mid ||a|| = 1\}$, $\rightsquigarrow \mathcal{S} = \mathcal{S}_0 \cup \mathcal{S}_1$.
- A question is to obtain a super version of the RSK correspondence (with symmetry) on super tableaux :
  - (Bonetti, Senato, Venezia, '88) : a super version using super Schensted's insertion
    - (La Scala, Nardozza, Senato, '06) : It is not symmetric
  - (Muth, '19) : super-RSK correspondence with symmetry
    - using Haiman's mixed insertion on super tableaux.
    - not related to the super plactic monoid and the super Littlewood-Richardson rule.

Signed alphabets

- **Signed alphabet** $(\mathcal{S}, ||.||)$:
  - $\mathcal{S}$ a finite or countable totally ordered set,
  - $||.|| : \mathcal{S} \longrightarrow \mathbb{Z}_2 = \{0, 1\}$ be any map,
  - $\mathcal{S}_0 = \{a \in \mathcal{S} \mid ||a|| = 0\}$ and $\mathcal{S}_1 = \{a \in \mathcal{S} \mid ||a|| = 1\}, \rightsquigarrow \mathcal{S} = \mathcal{S}_0 \cup \mathcal{S}_1$.
- A question is to obtain a super version of the RSK correspondence (with symmetry) on super tableaux :
  - (Bonetti, Senato, Venezia, '88) : a super version using super Schensted's insertion
    - (La Scala, Nardozza, Senato, '06) : It is not symmetric
  - (Muth, '19) : super-RSK correspondence with symmetry
    - using Haiman's mixed insertion on super tableaux.
    - not related to the super plactic monoid and the super Littlewood-Richardson rule.
- We introduce a **super-RSK correspondence** on super tableaux :

Signed alphabets

- **Signed alphabet** $(\mathcal{S}, ||.||)$:
  - $\mathcal{S}$ a finite or countable totally ordered set,
  - $||.|| : \mathcal{S} \longrightarrow \mathbb{Z}_2 = \{0, 1\}$ be any map,
  - $\mathcal{S}_0 = \{a \in \mathcal{S} \mid ||a|| = 0\}$ and $\mathcal{S}_1 = \{a \in \mathcal{S} \mid ||a|| = 1\}$, $\leadsto \mathcal{S} = \mathcal{S}_0 \cup \mathcal{S}_1$.
- A question is to obtain a super version of the RSK correspondence (with symmetry) on super tableaux :
  - (Bonetti, Senato, Venezia, '88) : a super version using super Schensted's insertion
    - (La Scala, Nardozza, Senato, '06) : It is not symmetric
  - (Muth, '19) : super-RSK correspondence with symmetry
    - using Haiman's mixed insertion on super tableaux.
    - not related to the super plactic monoid and the super Littlewood-Richardson rule.
- We introduce a **super-RSK correspondence** on super tableaux :
  - we prove the **symmetry** of the super-RSK correspondence,

Signed alphabets

- **Signed alphabet** $(\mathcal{S}, ||.||)$:
  - $\mathcal{S}$ a finite or countable totally ordered set,
  - $||.|| : \mathcal{S} \longrightarrow \mathbb{Z}_2 = \{0, 1\}$ be any map,
  - $\mathcal{S}_0 = \{a \in \mathcal{S} \mid ||a|| = 0\}$ and $\mathcal{S}_1 = \{a \in \mathcal{S} \mid ||a|| = 1\}$, $\rightsquigarrow \mathcal{S} = \mathcal{S}_0 \cup \mathcal{S}_1$.
- A question is to obtain a super version of the RSK correspondence (with symmetry) on super tableaux :
  - (Bonetti, Senato, Venezia, '88) : a super version using super Schensted's insertion
    - (La Scala, Nardozza, Senato, '06) : It is not symmetric
  - (Muth, '19) : super-RSK correspondence with symmetry
    - using Haiman's mixed insertion on super tableaux.
    - not related to the super plactic monoid and the super Littlewood-Richardson rule.
- We introduce a **super-RSK correspondence** on super tableaux :
  - we prove the **symmetry** of the super-RSK correspondence,
  - we obtain combinatorial versions of the **super Littlewood–Richardson rule**.

II. Super-RSK correspondence with symmetry

Super tableaux and insertion

Super tableaux and insertion

- $\mathrm{Yt}(\mathcal{S})$ set of **super tableaux** : $\mathcal{S}_0 = \{\text{even integers}\}$, $\mathcal{S}_1 = \{\text{odd integers}\}$

$$\mathrm{t} = \begin{array}{|c|c|c|c|c|c|c|} \hline 1 & 2 & 2 & 3 & 4 & 4 & 4 \\ \hline 1 & 3 & 5 & 7 & 8 & 8 & \multicolumn{1}{c}{} \\ \cline{1-6} 1 & 3 & 5 & 7 & \multicolumn{3}{c}{} \\ \cline{1-4} 4 & 4 & \multicolumn{5}{c}{} \\ \cline{1-2} \end{array}$$

Super tableaux and insertion

- $\mathrm{Yt}(\mathcal{S})$ set of **super tableaux** : $\mathcal{S}_0 = \{\text{even integers}\}$, $\mathcal{S}_1 = \{\text{odd integers}\}$

t =

1	2	2	3	4	4	4
1	3	5	7	8	8	
1	3	5	7			
4	4					

$R_{col}(t) = 4111\ 4332\ 552\ 773\ 84\ 84\ 4$

Super tableaux and insertion

- $\mathrm{Yt}(\mathcal{S})$ set of **super tableaux** : $\mathcal{S}_0 = \{\text{even integers}\}$, $\mathcal{S}_1 = \{\text{odd integers}\}$

t =

1	2	2	3	4	4	4
1	3	5	7	8	8	
1	3	5	7			
4	4					

$R_{col}(t) = 4111\ 4332\ 552\ 773\ 84\ 84\ 4$

- **Insertion on super tableaux**, (La Scala, Nardozza, Senato, '06) :

1	2	2	3	4	4	4
1	3	5	7	8	8	
1	3	5	7			
4	4					

$\leftsquigarrow 2 =$

1	2	2	2	4	4	4
1	3	5	7	8	8	
1	3	5	7			
3	4					
4						

Super tableaux and insertion

- $\mathrm{Yt}(\mathcal{S})$ set of **super tableaux** : $\mathcal{S}_0 = \{\text{even integers}\}$, $\mathcal{S}_1 = \{\text{odd integers}\}$

t =

1	2	2	3	4	4	4
1	3	5	7	8	8	
1	3	5	7			
4	4					

$R_{col}(t) \quad = 4111\ 4332\ 552\ 773\ 84\ 84\ 4$

- **Insertion on super tableaux**, (La Scala, Nardozza, Senato, '06) :

1	2	2	3	4	4	4
1	3	5	7	8	8	
1	3	5	7			
4	4					

$\leftsquigarrow 2 =$

1	2	2	2	4	4	4
1	3	5	7	8	8	
1	3	5	7			
3	4					
4						

- **Super tableau constructor** : $w = x_1 \ldots x_k$

$$T(w) \; := \; (\emptyset \leftsquigarrow w) \; = \; ((\ldots(\emptyset \leftsquigarrow x_1) \leftsquigarrow \; \ldots) \leftsquigarrow x_k).$$

Super tableaux and insertion

- $\mathrm{Yt}(\mathcal{S})$ set of **super tableaux** : $\mathcal{S}_0 = \{\text{even integers}\}$, $\mathcal{S}_1 = \{\text{odd integers}\}$

t =

1	2	2	3	4	4	4
1	3	5	7	8	8	
1	3	5	7			
4	4					

$R_{col}(t) = 4111\ 4332\ 552\ 773\ 84\ 84\ 4$

- **Insertion on super tableaux**, (La Scala, Nardozza, Senato, 06) :

1	2	2	3	4	4	4
1	3	5	7	8	8	
1	3	5	7			
4	4					

$\leftsquigarrow 2 =$

1	2	2	2	4	4	4
1	3	5	7	8	8	
1	3	5	7			
3	4					
4						

- **Super tableau constructor** : $w = x_1 \dots x_k$

$$T(w) := (\emptyset \leftsquigarrow w) = ((\dots(\emptyset \leftsquigarrow x_1) \leftsquigarrow \dots) \leftsquigarrow x_k).$$

Super tableaux and insertion

- $\mathrm{Yt}(\mathcal{S})$ set of **super tableaux** : $\mathcal{S}_0 = \{\text{even integers}\}$, $\mathcal{S}_1 = \{\text{odd integers}\}$

$$t = \begin{array}{|c|c|c|c|c|c|c|}\hline 1&2&2&3&4&4&4\\\hline 1&3&5&7&8&8\\\cline{1-6} 1&3&5&7\\\cline{1-4} 4&4\\\cline{1-2}\end{array} \qquad R_{col}(t) = 4111\ 4332\ 552\ 773\ 84\ 84\ 4$$

- **Insertion on super tableaux**, (La Scala, Nardozza, Senato, 06) :

$$\begin{array}{|c|c|c|c|c|c|c|}\hline 1&2&2&3&4&4&4\\\hline 1&3&5&7&8&8\\\cline{1-6} 1&3&5&7\\\cline{1-4} 4&4\\\cline{1-2}\end{array} \leftsquigarrow 2 = \begin{array}{|c|c|c|c|c|c|c|}\hline 1&2&2&2&4&4&4\\\hline 1&3&5&7&8&8\\\cline{1-6} 1&3&5&7\\\cline{1-4} 3&4\\\cline{1-2} 4\\\cline{1-1}\end{array}$$

- **Super tableau constructor** : $w = x_1 \dots x_k$

$$T(w) := (\emptyset \leftsquigarrow w) = ((\dots(\emptyset \leftsquigarrow x_1) \leftsquigarrow \dots) \leftsquigarrow x_k).$$

- **Insertion product** :

$$t \star t' := (t \leftsquigarrow R_{col}(t'))$$

Super-RSK correspondence with symmetry

Super-RSK correspondence with symmetry

- **Signed two rowed array**.

  $\mathcal{S}_0 = \mathcal{S}_1' = \{\text{even numbers}\}$, $\mathcal{S}_1 = \mathcal{S}_0' = \{\text{odd numbers}\}$:

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \\ 3 & 2 & 1 & 2 & 4 & 3 & 1 & 2 \end{pmatrix}$$

Super-RSK correspondence with symmetry

- **Signed two rowed array.**

  $\mathcal{S}_0 = \mathcal{S}_1' = \{\text{even numbers}\}, \mathcal{S}_1 = \mathcal{S}_0' = \{\text{odd numbers}\}$:

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \\ 3 & 2 & 1 & 2 & 4 & 3 & 1 & 2 \end{pmatrix}$$

- **Super-RSK correspondence** :

$$(\emptyset\ ,\ \emptyset),\quad \left(\begin{array}{c}3\end{array},\ \begin{array}{c}1\end{array}\right),\quad \left(\begin{array}{c}2\\3\end{array},\ \begin{array}{c}1\\1\end{array}\right),\quad \left(\begin{array}{c}1\\2\\3\end{array},\ \begin{array}{c}1\\1\\2\end{array}\right),\quad \left(\begin{array}{ll}1&2\\2&\\3&\end{array},\ \begin{array}{ll}1&2\\1&\\2&\end{array}\right),$$

$$\left(\begin{array}{lll}1&2&4\\2&&\\3&&\end{array},\ \begin{array}{lll}1&2&3\\1&&\\2&&\end{array}\right),\quad \left(\begin{array}{lll}1&2&3\\2&4&\\3&&\end{array},\ \begin{array}{lll}1&2&3\\1&3&\\2&&\end{array}\right),\quad \left(\begin{array}{lll}1&1&3\\2&4&\\2&&\\3&&\end{array},\ \begin{array}{lll}1&2&3\\1&3&\\2&&\\4&&\end{array}\right),$$

$$\left(T(W) = \begin{array}{lll}1&1&2\\2&3&\\2&4&\\3&&\end{array},\ Q(W) = \begin{array}{lll}1&2&3\\1&3&\\2&4&\\4&&\end{array}\right).$$

Super-RSK correspondence with symmetry

- **Signed two rowed array.**

  $\mathcal{S}_0 = \mathcal{S}_1' = \{\text{even numbers}\}, \mathcal{S}_1 = \mathcal{S}_0' = \{\text{odd numbers}\}$:

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \\ 3 & 2 & 1 & 2 & 4 & 3 & 1 & 2 \end{pmatrix}$$

- **Super-RSK correspondence** :

$$(\emptyset\ ,\ \emptyset),\quad \left(\boxed{3}\ ,\ \boxed{1}\right),\quad \left(\begin{array}{|c|}\hline 2\\ \hline 3\\ \hline\end{array}\ ,\ \begin{array}{|c|}\hline 1\\ \hline 1\\ \hline\end{array}\right),\quad \left(\begin{array}{|c|}\hline 1\\ \hline 2\\ \hline 3\\ \hline\end{array}\ ,\ \begin{array}{|c|}\hline 1\\ \hline 1\\ \hline 2\\ \hline\end{array}\right),\quad \left(\begin{array}{ll} 1 & 2 \\ 2 & \\ 3 & \end{array}\ ,\ \begin{array}{ll} 1 & 2 \\ 1 & \\ 2 & \end{array}\right),$$

$$\left(\begin{array}{lll} 1 & 2 & 4 \\ 2 & & \\ 3 & & \end{array}\ ,\ \begin{array}{lll} 1 & 2 & 3 \\ 1 & & \\ 2 & & \end{array}\right),\quad \left(\begin{array}{lll} 1 & 2 & 3 \\ 2 & 4 & \\ 3 & & \end{array}\ ,\ \begin{array}{lll} 1 & 2 & 3 \\ 1 & 3 & \\ 2 & & \end{array}\right),\quad \left(\begin{array}{lll} 1 & 1 & 3 \\ 2 & 4 & \\ 2 & & \\ 3 & & \end{array}\ ,\ \begin{array}{lll} 1 & 2 & 3 \\ 1 & 3 & \\ 2 & & \\ 4 & & \end{array}\right),$$

$$\left(T(W) = \begin{array}{lll} 1 & 1 & 2 \\ 2 & 3 & \\ 2 & 4 & \\ 3 & & \end{array}\ ,\ Q(W) = \begin{array}{lll} 1 & 2 & 3 \\ 1 & 3 & \\ 2 & 4 & \\ 4 & & \end{array}\right).$$

Theorem (H. '23). There is a one-to-one correspondence :

$$W \overset{\text{RSK}}{\longleftrightarrow} (T(W), Q(W))$$

Super-RSK correspondence with symmetry

- **Signed two rowed array**.

  $\mathcal{S}_0 = \mathcal{S}'_1 = \{\text{even numbers}\}, \mathcal{S}_1 = \mathcal{S}'_0 = \{\text{odd numbers}\}$:

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \\ 3 & 2 & 1 & 2 & 4 & 3 & 1 & 2 \end{pmatrix}$$

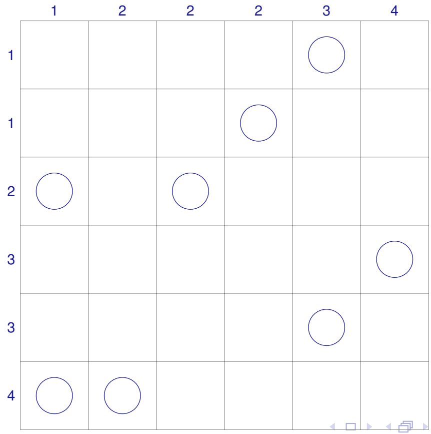
Super-RSK correspondence with symmetry

- **Signed two rowed array**.

  $\mathcal{S}_0 = \mathcal{S}'_1 = \{\text{even numbers}\}, \mathcal{S}_1 = \mathcal{S}'_0 = \{\text{odd numbers}\}$:

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \\ 3 & 2 & 1 & 2 & 4 & 3 & 1 & 2 \end{pmatrix}$$

- **Matrix-ball construction :**



Super-RSK correspondence with symmetry

- **Signed two rowed array.**

  $\mathcal{S}_0 = \mathcal{S}_1' = \{\text{even numbers}\}, \mathcal{S}_1 = \mathcal{S}_0' = \{\text{odd numbers}\}$:

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \\ 3 & 2 & 1 & 2 & 4 & 3 & 1 & 2 \end{pmatrix}$$

- **Matrix-ball construction :**

- **Signed two rowed array**.

  $\mathcal{S}_0 = \mathcal{S}_1' = \{\text{even numbers}\}, \mathcal{S}_1 = \mathcal{S}_0' = \{\text{odd numbers}\}$:

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \\ 3 & 2 & 1 & 2 & 4 & 3 & 1 & 2 \end{pmatrix}$$

- **Matrix-ball construction :**

	1	2	2	2	3	4
1					1	
1				1		
2	1		2			
3						3
3					3	
4	2	3				

Super-RSK correspondence with symmetry

- **Signed two rowed array**.

  $\mathcal{S}_0 = \mathcal{S}_1' = \{\text{even numbers}\}, \mathcal{S}_1 = \mathcal{S}_0' = \{\text{odd numbers}\}$:

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \\ 3 & 2 & 1 & 2 & 4 & 3 & 1 & 2 \end{pmatrix}$$

- **Matrix-ball construction :**

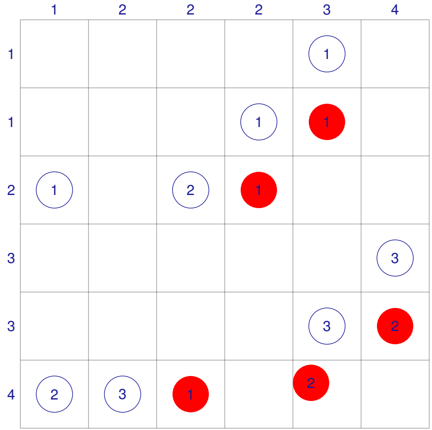
Super-RSK correspondence with symmetry

- **Signed two rowed array**.

  $\mathcal{S}_0 = \mathcal{S}_1' = \{\text{even numbers}\}, \mathcal{S}_1 = \mathcal{S}_0' = \{\text{odd numbers}\}$:

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \\ 3 & 2 & 1 & 2 & 4 & 3 & 1 & 2 \end{pmatrix}$$

- **Matrix-ball construction :**



Super-RSK correspondence with symmetry

- **Signed two rowed array**.

  $\mathcal{S}_0 = \mathcal{S}_1' = \{\text{even numbers}\}$, $\mathcal{S}_1 = \mathcal{S}_0' = \{\text{odd numbers}\}$:

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \\ 3 & 2 & 1 & 2 & 4 & 3 & 1 & 2 \end{pmatrix}$$

- **Matrix-ball construction :**

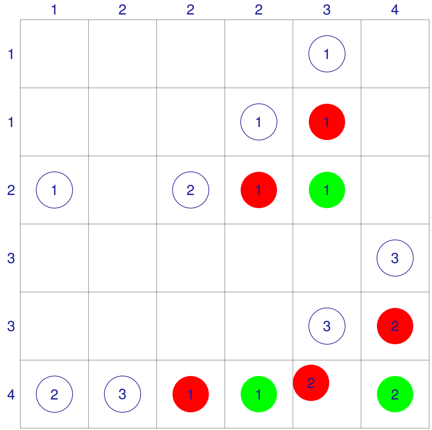
Super-RSK correspondence with symmetry

- **Signed two rowed array.**

  $\mathcal{S}_0 = \mathcal{S}_1' = \{\text{even numbers}\}, \mathcal{S}_1 = \mathcal{S}_0' = \{\text{odd numbers}\}$:

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \\ 3 & 2 & 1 & 2 & 4 & 3 & 1 & 2 \end{pmatrix}$$

- **Matrix-ball construction :**



Super-RSK correspondence with symmetry

- **Signed two-rowed array**.

  $\mathcal{S}_0 = \mathcal{S}_1' = \{\text{even numbers}\}, \mathcal{S}_1 = \mathcal{S}_0' = \{\text{odd numbers}\}$:

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \\ 3 & 2 & 1 & 2 & 4 & 3 & 1 & 2 \end{pmatrix}$$

- **Matrix-ball construction :**

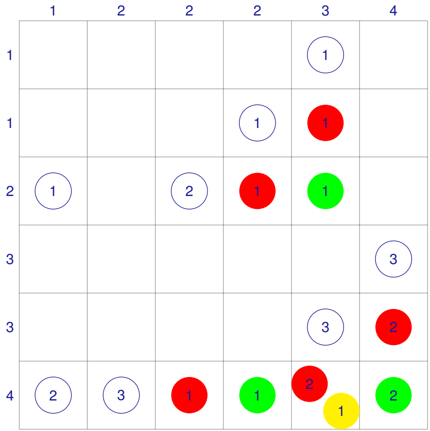
Super-RSK correspondence with symmetry

- **Signed two-rowed array**.

  $\mathcal{S}_0 = \mathcal{S}'_1 = \{\text{even numbers}\}, \mathcal{S}_1 = \mathcal{S}'_0 = \{\text{odd numbers}\}$:

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \\ 3 & 2 & 1 & 2 & 4 & 3 & 1 & 2 \end{pmatrix}$$

- **Matrix-ball construction :**



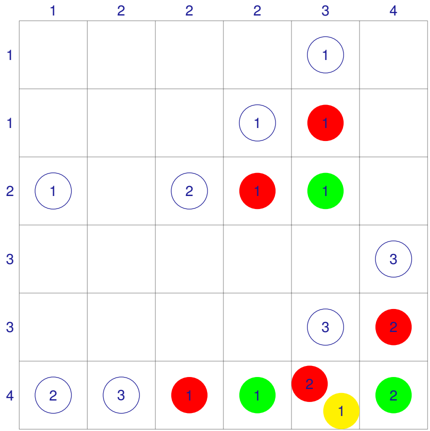
Super-RSK correspondence with symmetry

- **Signed two-rowed array.**

  $\mathcal{S}_0 = \mathcal{S}_1' = \{\text{even numbers}\}, \mathcal{S}_1 = \mathcal{S}_0' = \{\text{odd numbers}\}$:

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \\ 3 & 2 & 1 & 2 & 4 & 3 & 1 & 2 \end{pmatrix}$$

- **Matrix-ball construction :**



$$\left(T(W) = \begin{array}{|c|c|c|} \hline 1 & 1 & 2 \\ \hline 2 & 3 \\ \cline{1-2} 2 & 4 \\ \cline{1-2} 3 \\ \cline{1-1} \end{array}, \quad Q(W) = \begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 1 & 3 \\ \cline{1-2} 2 & 4 \\ \cline{1-2} 4 \\ \cline{1-1} \end{array} \right)$$

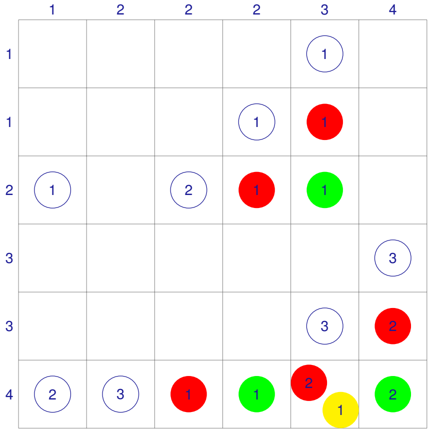
Super-RSK correspondence with symmetry

- **Signed two rowed array**.

  $\mathcal{S}_0 = \mathcal{S}'_1 = \{\text{even numbers}\}, \mathcal{S}_1 = \mathcal{S}'_0 = \{\text{odd numbers}\}$:

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \\ 3 & 2 & 1 & 2 & 4 & 3 & 1 & 2 \end{pmatrix}$$

- **Matrix-ball construction :**



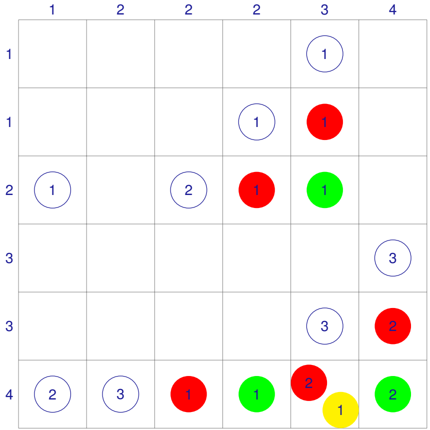
Super-RSK correspondence with symmetry

- **Signed two rowed array**.

  $\mathcal{S}_0 = \mathcal{S}'_1 = \{\text{even numbers}\}, \mathcal{S}_1 = \mathcal{S}'_0 = \{\text{odd numbers}\}$:

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \\ 3 & 2 & 1 & 2 & 4 & 3 & 1 & 2 \end{pmatrix}$$

- **Matrix-ball construction :**



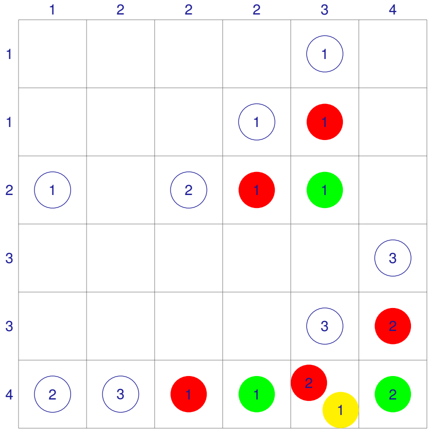
$$W^{\text{inv}} = \begin{pmatrix} 1 & 1 & 2 & 2 & 2 & 3 & 3 & 4 \\ 2 & 4 & 4 & 2 & 1 & 1 & 3 & 3 \end{pmatrix}$$

- **Signed two rowed array**.

  $\mathcal{S}_0 = \mathcal{S}_1' = \{\text{even numbers}\}, \mathcal{S}_1 = \mathcal{S}_0' = \{\text{odd numbers}\}$:

$$W = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 \\ 3 & 2 & 1 & 2 & 4 & 3 & 1 & 2 \end{pmatrix}$$

- **Matrix-ball construction :**



$$W^{\text{inv}} = \begin{pmatrix} 1 & 1 & 2 & 2 & 2 & 3 & 3 & 4 \\ 2 & 4 & 4 & 2 & 1 & 1 & 3 & 3 \end{pmatrix}$$

$$\updownarrow$$

$$\left(Q(W) = \begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 1 & 3 \\ \cline{1-2} 2 & 4 \\ \cline{1-2} 4 \\ \cline{1-1} \end{array}, \; T(W) = \begin{array}{|c|c|c|} \hline 1 & 1 & 2 \\ \hline 2 & 3 \\ \cline{1-2} 2 & 4 \\ \cline{1-2} 3 \\ \cline{1-1} \end{array} \right)$$

Super-RSK correspondence with symmetry

Super-RSK correspondence with symmetry

Theorem (H. '23). (**The symmetry property**)

$$W \overset{RSK}{\longleftrightarrow} (T(W), Q(W)) \iff W^{\text{inv}} \overset{RSK}{\longleftrightarrow} (Q(W), T(W)).$$

Super-RSK correspondence with symmetry

Super-RSK correspondence with symmetry

- We introduce a super-RSK correspondence on super tableaux with symmetry.

Super-RSK correspondence with symmetry

- We introduce a super-RSK correspondence on super tableaux with symmetry.
- We obtain combinatorial interpretations of the **super Littlewood–Richardson rule** using **super Schur functions**.

Super-RSK correspondence with symmetry

- We introduce a super-RSK correspondence on super tableaux with symmetry.
- We obtain combinatorial interpretations of the **super Littlewood–Richardson rule** using **super Schur functions**.
- We obtain bijective proofs of super **Gauchy identities** on signed alphabets.

Super-RSK correspondence with symmetry

- We introduce a super-RSK correspondence on super tableaux with symmetry.
- We obtain combinatorial interpretations of the **super Littlewood–Richardson rule** using **super Schur functions**.
- We obtain bijective proofs of super **Gauchy identities** on signed alphabets.

  - **A super Littlewood–Richardson type rule**, to appear in Springer Proceedings in Mathematics and Statistics (PROMS), 2024.
  - **A super-RSK correspondence with symmetry and the super Littlewood–Richardson rule for super tableaux**, arXiv :2206.15451, submitted, 2023.
  - **Super jeu de taquin and combinatorics of super tableaux of type A**, International Journal of Algebra and Computation, arxiv :2105.07819, 2022.

Thank you for your attention !