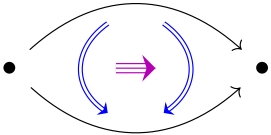
Different approaches to opetopes

Louise LECLERC

Joint work with :
Pierre Louis CURIEN

Higher categorical setting



Globular

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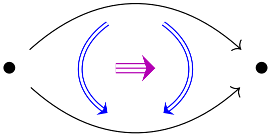

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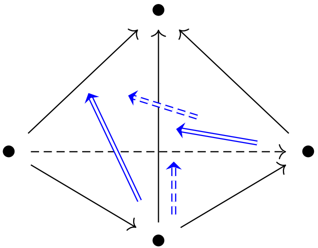
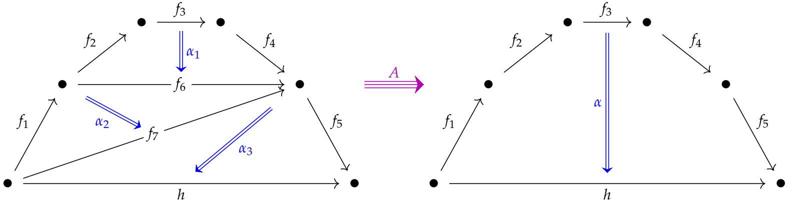


Simplicial

Higher categorical setting



Opetopic

What are opetopes ?

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0-opetope

What are opetopes ?

$$\bullet$$

0-opetope

$$\bullet \longrightarrow \bullet$$

1-opetope

What are opetopes ?

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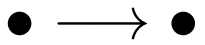

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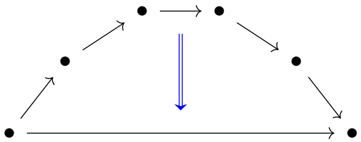
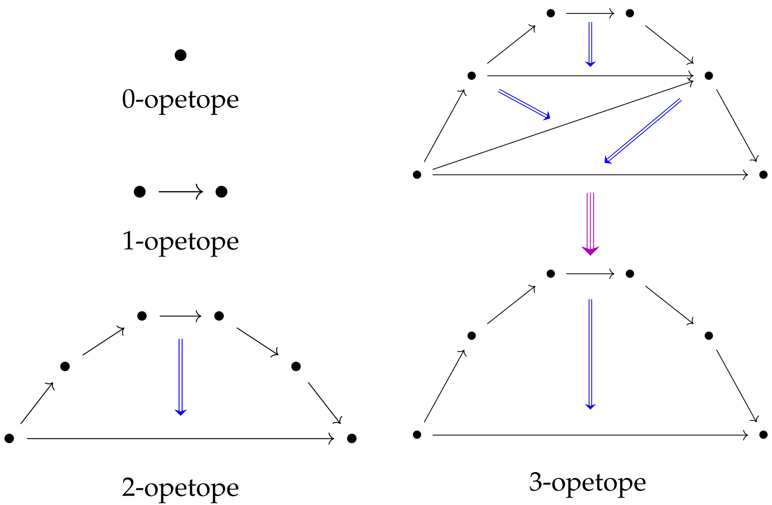


2-opetope

What are opetopes ?



What are opetopes ?

Opetope = ope(ration + poly)tope

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Kind of oriented polytope

Many sources, one target

Poset

Marek ZAWADOWSKI and Amar HADZIHASANOVIC (independently).

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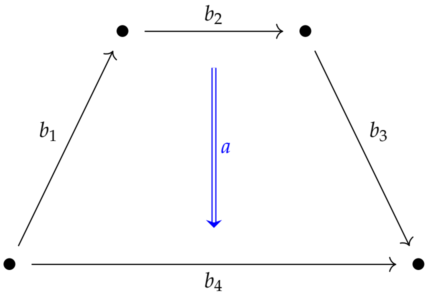
$b_1 \prec^- a$

$b_2 \prec^- a$

$b_3 \prec^- a$

$b_4 \prec^+ a$



Positive-to-one poset

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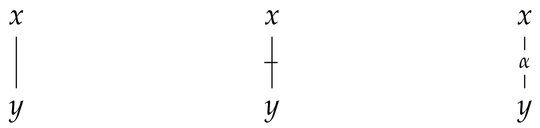
We let

$$\delta(x) := \{y \in P \mid y \prec^- x\} \qquad \gamma(x) := \{y \in P \mid y \prec^+ x\}$$

Positive-to-one poset



$y \prec^- x$ $y \prec^+ x$ $y \prec^\alpha x$

Dendritic face complex (CURIEN, HADZIHASANOVIC)

A Positive-to-one poset P such that :

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  P has a greatest element.

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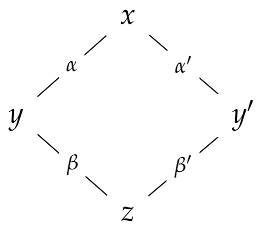

Dendritic face complex (Curien, Hadzihasanovic)

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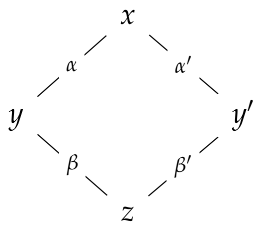
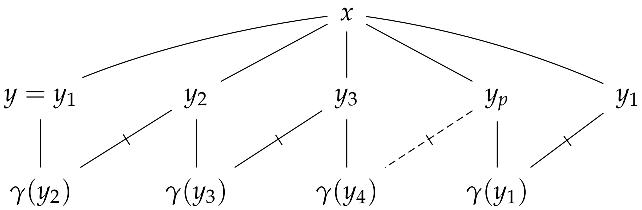
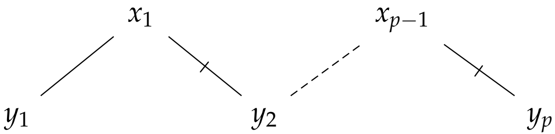




Positive opetopic cardinal (ZAWADOWSKI)



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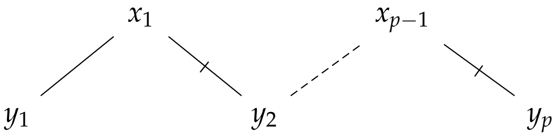

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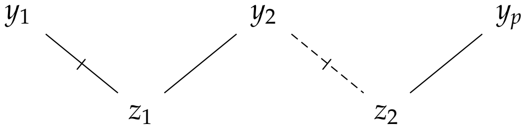

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- (*pencil linearity*)

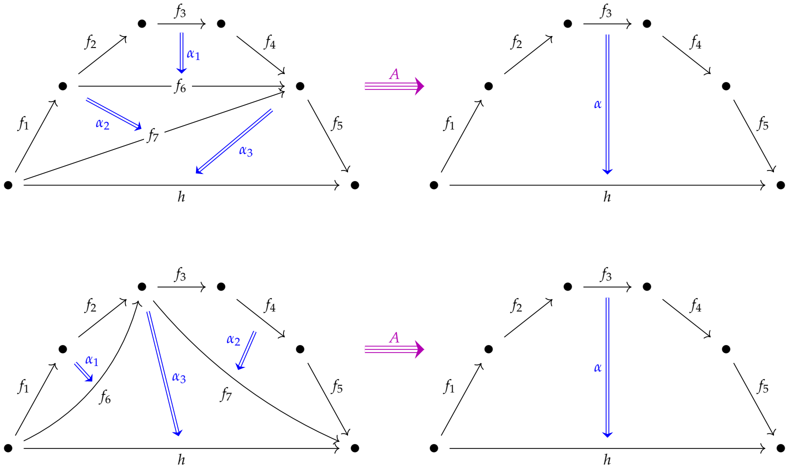
$$\{x \in S_k \mid y = \gamma(x)\} \qquad \{x \in S_k \mid y \in \delta(x)\}$$

linearly ordered

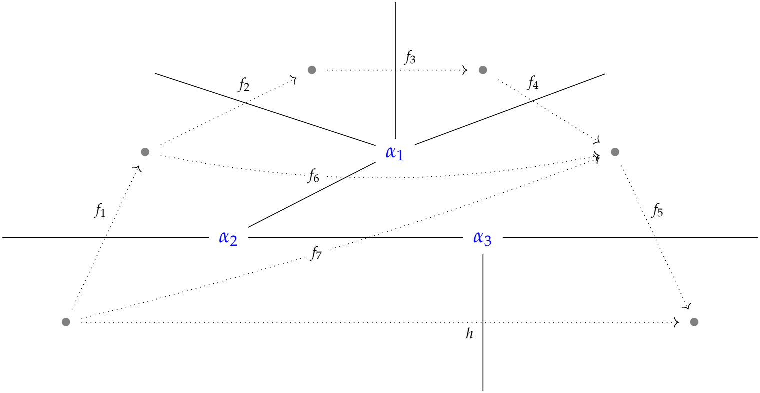
Epiphytes (with CURIEN)

https://mezzopieno-news.tumblr.com/post/173475498826/il-bialbero-rinato-fiorisce-la-natura-risponde

Epiphytes



Epiphytes



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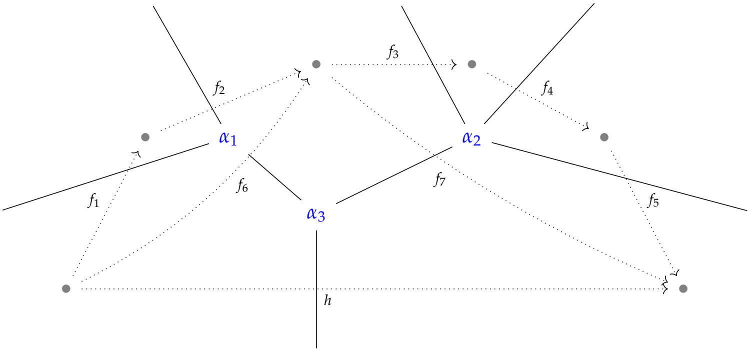
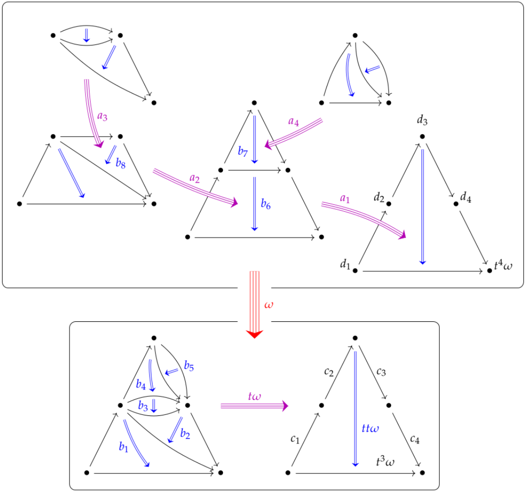
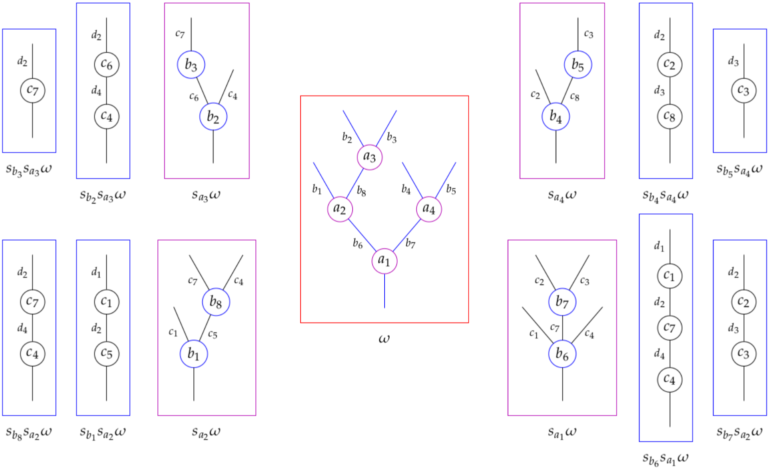


A 4-opetope


a3
b8
a2
b7
a4
b6
a1
d3
d2
d4
d1
t4ω
ω
b5
b4
b3
b2
b1
tω
c2
c3
c1
ttω
c4
t3ω

And its 4-epiphyte



Rooted tree

A rooted tree T is:

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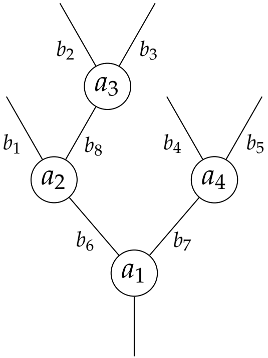
with a root $\rho(T)$ s.t.

$$\forall a \in T^\bullet, \exists! \left(a = a_0 \succ_{b_1} a_1 \succ_{b_1} \cdots \succ_{b_p} a_p = \rho(T) \right)$$

Rooted tree



Epiphytes

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  - For every node a:
    a n-epiphyte $s_a\omega$ s.t. $(s_a\omega)^\bullet = A(a)$.
    $\forall a \prec_b a', \forall (b', c) \in (s_{a'}\omega)^|, s_b s_a \omega = t s_{a'} \omega$

Target

For ω a $(n+1)$-epiphyte, $t\omega$ is a n-epiphyte :

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- Root : $\kappa(\rho(\omega), \rho(s_{\rho(\omega)}\omega))$

Theorem

There are equivalences of categories:

$$\textbf{Epiphytes} \underset{\simeq}{\rightleftarrows} \textbf{DFC}^{+} \underset{\simeq}{\rightleftarrows} \textbf{POC}$$

What are opetopes (v2) ?

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0-opetope

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$\bullet$

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$\bullet \longrightarrow \bullet$

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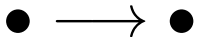

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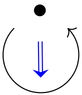

2-opetope

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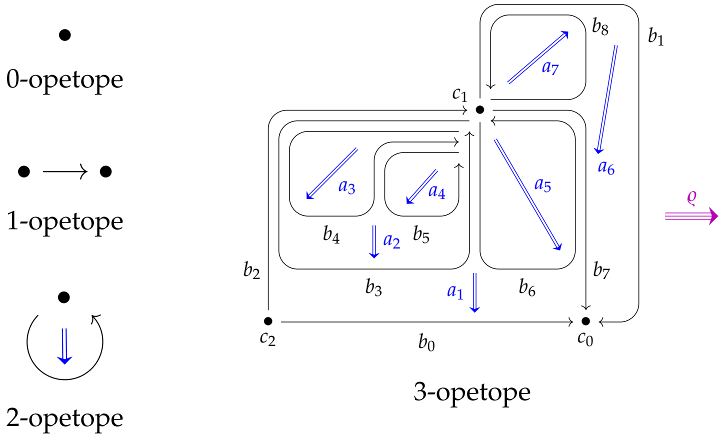

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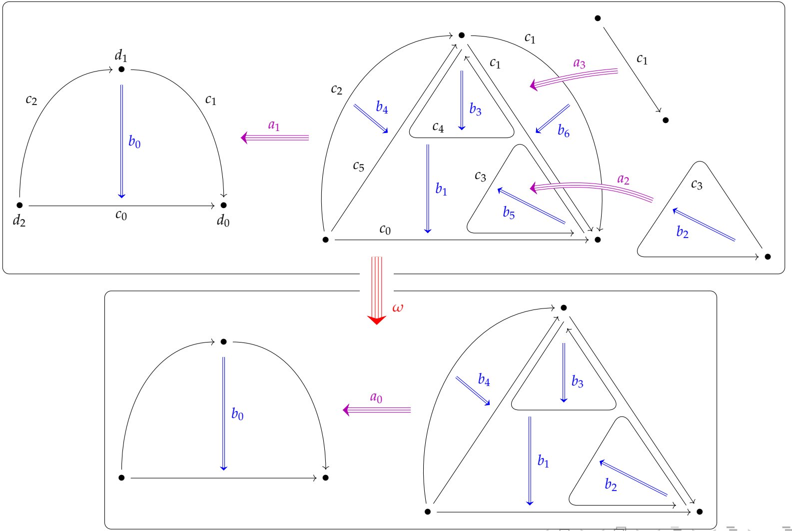

poset-like approach

Generalizing DFC^+ to DFC.

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We add $\prec^{\circ}$.

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(*loop filing condition*)

poset-like approach

Generalizing DFC$^+$ to DFC.

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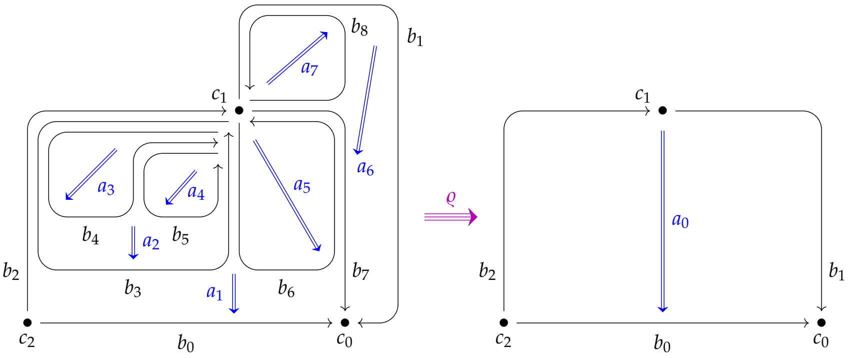
(*loop filing condition*)

Refinement of (*oriented thinness*).

poset like approach



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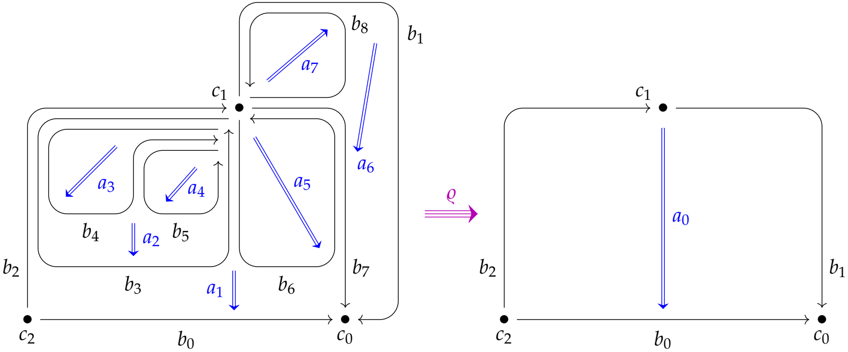
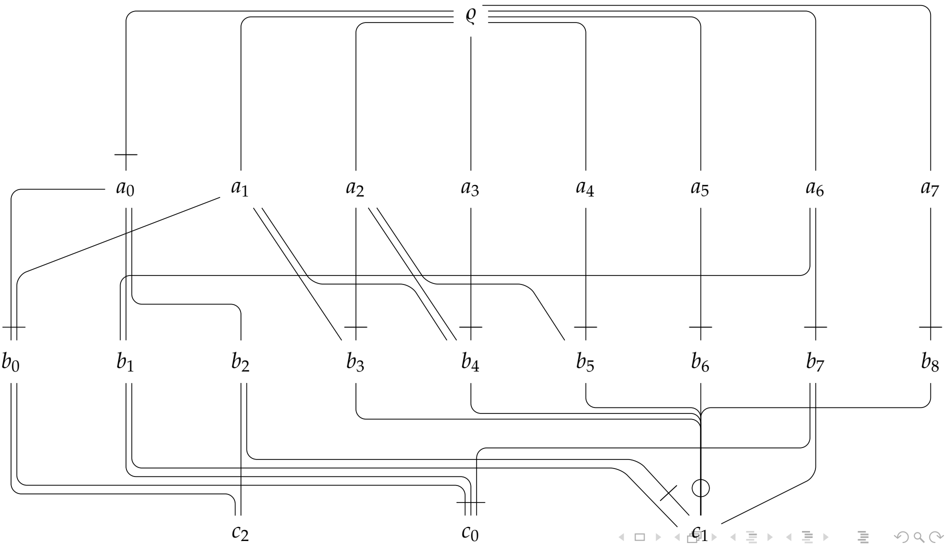


We should order loops !

poset like approach



”astronomical” approach (JKBM)

We generalise rooted trees.

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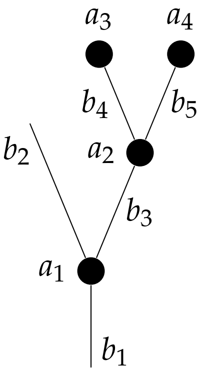

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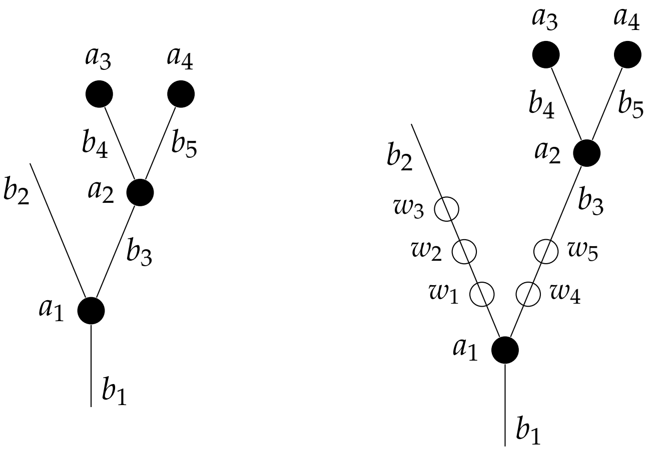

Constellations

A *constellation* $C : T \to U$ is:

a subdivision T' of T.

a bijection $\sigma_\bullet$: $\textsf{blackdots}(T') \to \textsf{leaves}(U)$.

a bijection $\sigma_\circ$: $\textsf{whitedots}(T') \to \textsf{nulldots}(U)$.

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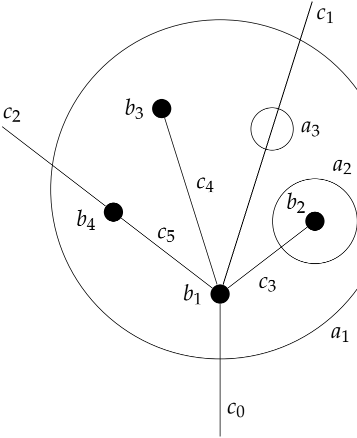
s.t. $\sigma = \sigma_\bullet + \sigma_\circ$ satisfies the *kernel rule*:

For $x \in \mathsf{dots}(U)$, the nodes $\{t \in \mathsf{dots}(T') \mid \sigma(t) \sqsubseteq x\}$ span a connected full subgraph of T'.

Constellations



zooms complexes

A *zoom complex* is a sequence

$$T_0 \xrightarrow{C_1} T_1 \xrightarrow{C_2} \cdots \xrightarrow{C_n} T_n$$

of rooted trees and constellations.

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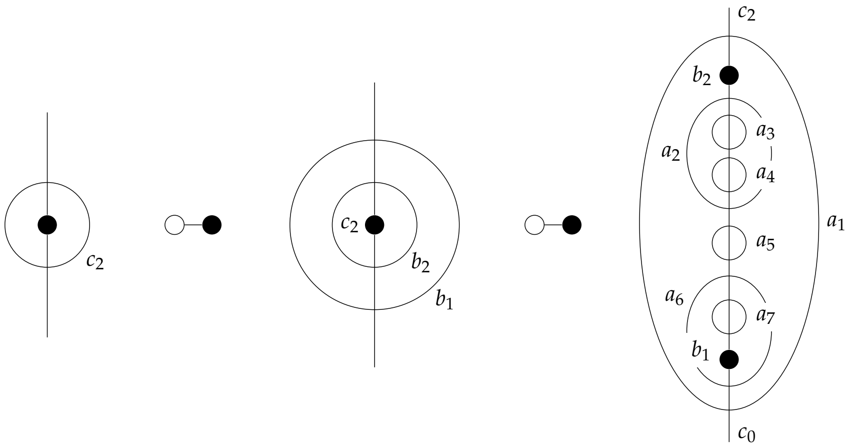
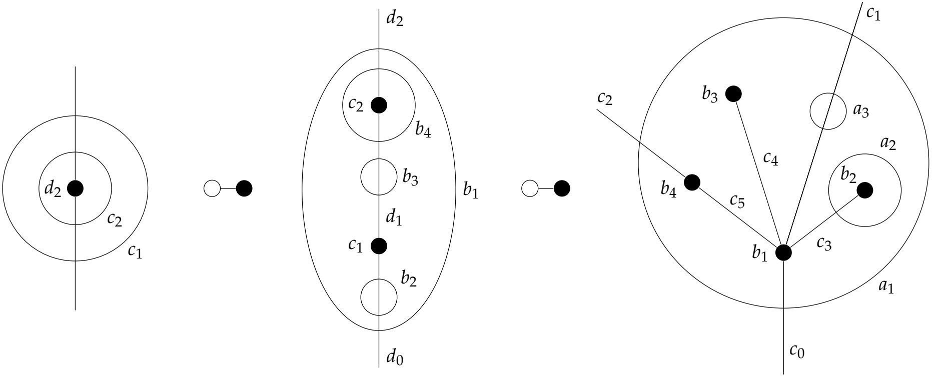
An opetope is a zoom complex starting by two "unit tree" and one linear tree.

Exemple 1



Exemple 2


d_2
c_2
c_1
d_2
c_2
b_4
b_3
b_1
d_1
c_1
b_2
d_0
c_1
c_2
b_3
a_3
a_2
c_4
b_2
b_4
c_5
b_1
c_3
a_1
c_0

Theorem

There is an equivalence of groupoids

$$\mathbf{DFC}^{\mathsf{core}} \simeq \mathbf{JKBM}$$

"Epiphytes" approach

Work in progress !

Thank you for your attention !

Any question ?